

# CORE SYSTEMS AND THE GROWTH OF HUMAN KNOWLEDGE: NATURAL GEOMETRY

■ ELIZABETH S. SPELKE

## Cultivating seeds of human knowledge: natural geometry

Together with the mastery of reading and the achievement of literacy (Dehaene, this volume), the mastery of elementary mathematics is the most important achievement of the early school years. A basic understanding of number and geometry is fundamental to almost all the activities of modern humans, from trade and economics to measurement and technology to science and the arts. Like reading, moreover, mathematics poses challenges for many students, who may experience difficulty learning and reasoning about numbers, maps and graphs throughout their lives. Fostering education in mathematics therefore is a central focus of efforts to improve the education of all children. Here, I focus on the foundations of an aspect of mathematical learning that has received somewhat less attention than number and arithmetic: the development of geometrical intuitions.

As a formal subject matter, Euclidean geometry typically is not introduced into mathematics education until the end of the elementary school years, because instruction in its subject matter poses challenges. Most topics in the elementary school curriculum can be connected to meaningful and engaging activities: numbers and counting can be taught in the context of board games (Siegler & Ramani, 2011), and reading in the context of stories, plays, and songs. In contrast, the objects of geometry – dimensionless points, lines of infinite extent, and ideal forms – cannot be seen nor acted upon. Perhaps as a result, formal geometry often enters the mathematics curriculum only around adolescence, where it is presented as a set of exercises in logical theorem proving.

Nevertheless, educated adults have clear geometrical intuitions, and these intuitions stand at the foundations of a body of knowledge that has long been central to human thought and action. Both the naturalness and the elusiveness of the points, lines and planes of Euclidean geometry led Socrates to argue that geometry cannot be taught or learned at all, but only recollected from ancestral memories (Plato, ca. 380 b.c.). Recent research suggests, however, that Euclidean geometry develops over the course of childhood, as a product of systems that emerge in human infancy and guide a rich array of activities that engage children long before they learn to read or count.

## Developing geometrical intuitions

Evidence for the late emergence of Euclidean geometrical intuitions comes from studies of the development of three abilities: the ability to navigate by a purely geometric map, the ability to reason about the properties of triangles on a planar or curved surface, and the ability to reflect on the fundamental properties of points and lines. These abilities have been assessed through research on adults and children living in two markedly different kinds of settings: urban, industrialized societies in North America and Europe, and scattered villages in a remote region of the Amazon. I will briefly describe the findings of these studies.

First, navigation by geometric maps has been studied both in adults and in children as young as 4 years of age (Dehaene *et al.*, 2006; Shusterman *et al.*, 2008). In these studies, participants are shown an array of three opaque containers in a specific geometric arrangement (a triangle or an unequally spaced line) and are encouraged to place or find an object in one specific container. To locate the correct container, participants are turned away from the display and shown a schematic picture of an overhead view of the array – three discs arranged in a similar triangle or line – with a star marking the target location (see Figure 1, p. 245). At all ages, children and adults used *distance* relationships on this map to locate the target in the navigable array, showing that even the 4-year-old children understood the task and were motivated to perform it. Adults in both cultures also located the target by analyzing two other fundamental properties of Euclidean geometry: *angle* (the information that distinguishes corners of a triangle that differ in size) and *sense* (the information that distinguishes a form from its mirror image; Dehaene *et al.*, 2006). Four-year-old children, in contrast, only extracted distance relations from the map (Shusterman *et al.*, 2008). Further studies revealed that children begin to use angle information at 6 years (Spelke *et al.*, in press) and sense information at adolescence (Hyde *et al.*, 2011). These studies reveal a slow, progressive development of sensitivity to Euclidean maps.

Second, geometrical reasoning has been studied by assessing the abilities of adults and children to find and reproduce the third corner of a triangle (Izard *et al.*, 2011a). For this task, participants were presented with animated images on a computer screen, depicting a large extended surface that was either flat or spherical. In an accompanying narration, the surface was described as a land with three small villages, connected by straight paths (see Figure 2a, p. 246). Two dots, each with two arrows, then appeared on the lower sides of the screen. The dots were described as villages, and the arrows were described as the beginnings of the paths that connected the villages to each other and to a third village, not shown. Participants were asked to

indicate both the location of the third village and the angle at which the two paths met at that village. When reasoning about paths on the plane, the judgments of adults and older children in both cultures accorded with the principles of Euclidean geometry: they produced an angle that, together with the two visible angles on the screen, summed to a value very close to 180 degrees, independent of the distance between the two visible points or the area of the triangle thus formed. When reasoning about paths on the sphere, these older children and adults adapted their responses in the appropriate direction, producing angles whose size increased with increases in the triangle's area (although adults and children alike tended to underestimate the degree to which this angle should increase on the sphere). In contrast, 6-year-old children showed no ability to distinguish between the planar and spherical worlds. Contrary to the basic principles of Euclidean plane geometry, the angles that they produced were independent of the angles visible on the screen and varied with the distances between the points on the screen. For the youngest children, reasoning about navigable paths in this virtual world did not accord with the basic, Euclidean properties of planar triangles.

Intuitions about points and lines were elicited from the same adults and children who participated in the virtual navigation task (Izard *et al.*, 2011a). Participants again were shown the two textured surfaces on the computer screen: the planar surface for one block of trials and the spherical surface for the other block of trials. Then the computer image approached the surface until all information for surface curvature was removed, and subjects were presented with simple displays containing a few points or short line segments, accompanied by yes/no questions (see Figure 2b, p. 246). Some of these questions probed properties of points and lines that are true of both plane and spherical geometry (for example, can a straight line be drawn between any two points? between any three points?). Other questions probed properties of points and lines that are true of one of these geometries but not the other (for example, are there pairs of lines that never cross? that cross more than once?).

As in the previous study, adults and children aged 10 and above answered these questions systematically. When questions were asked of the planar surface, their performance accorded with Euclidean geometry: participants judged with high consistency that some lines will never cross on the plane and none will cross more than once. Participants also tended to adjust the latter judgment in the case of the sphere, noting that straight lines on the sphere can cross twice (although participants in both cultures tended mistakenly to judge that some straight lines on the sphere will never cross). Once

again, the performance of 6-year-old children contrasted with that of their elders: children gave consistent judgments about some of the properties that are common to points and lines on a plane or sphere, but they did so far less systematically than did adults and older children, and they failed to differentiate between planar and spherical surfaces in their judgments.

In summary, adults and older children show knowledge of abstract, Euclidean geometry in three tasks: a map task in which they must extract information about length, angle and sense relations from a 2D picture and apply this information to a 3D navigable array, a triangle completion task in which they must reason about the properties of paths connecting villages on a flat or curved surface, and a test that elicits pure intuitions about the basic properties of dimensionless points and straight, endless lines. Before 6 years of age, however, children share only a small part of these abilities. When and how does Euclidean geometry become natural and intuitive to children, and what experiences foster this developmental change?

### **Discovering the sources of geometrical intuitions**

In the rest of this chapter, I explore an old idea: When humans learn and practice formal geometry, we take capacities of the mind and brain that evolved in our species and developed in infancy and early childhood to serve other functions, and we harness them for this new purpose. Cognitive psychologists and neuroscientists can discover those systems, and their functions, through four different strands of comparative research. One research effort compares the geometrical abilities of children of different ages, from infancy to maturity. A second research effort compares the abilities of different animal species, from humans' close primate relatives to more distant vertebrates and even invertebrates. A third research effort compares the abilities of humans living in different cultures, with differing access to education and to cultural products such as maps and rulers, and also humans who differ in their spatial abilities. A fourth research effort traces geometrical cognition across different levels of analysis, from genes to neurons to brain systems to learning and behavior.

I believe that these four strands of research converge to reveal two core systems at the foundations of human knowledge of geometry. One is a system for representing the large-scale navigable layout: a system that humans and animals use to specify their own location within such a layout. The other is a system for representing small-scale manipulable objects and forms: a system that humans and animals use to recognize and categorize objects of significant kinds. Humans discover the system of abstract, Euclidean geometry, I believe, by productively combining representations from these

two systems. Those combinations, in turn, depend on a host of uniquely human, symbolic devices.

In the rest of this chapter, I describe each of the two systems of core knowledge of geometry in turn, focusing both on its richness and on its limits. Then I turn to a third system that children and adults use to combine representations from these systems, and that therefore may foster children's construction of a more abstract geometry: the system of natural language.

### **Geometry for navigation**

'Geometry', the measurement of the earth, is aptly named. The core system of core knowledge of geometry is a system by which navigating humans and other animals compute their own positions, and those of significant objects, by measuring properties of the surrounding terrain. An experiment conducted on 18- to 24-month-old children, serves to introduce this system (Hermer & Spelke, 1996). Children were brought into a closed unfurnished rectangular room with four corner panels at which toys could be hidden (see Figure 3a, p. 247). Because the room was uniformly colored and illuminated, only the relative lengths of its walls broke its four-fold symmetry; no information distinguished any direction from its diagonal opposite. Children stood at the room's center and watched as a toy was hidden at one corner. Then they were lifted and turned slowly with eyes covered until they were disoriented. Finally they were released and encouraged to search for the toy. Children confined their search to two of the room's corners: the correct location and the opposite, geometrically congruent location. This finding provides evidence that children's navigation was guided in some way by the shape of their surroundings.

Although full disorientation is a rare event during human navigation, experiments using reorientation tasks are highly valuable, because they reveal the information about the environment that navigators encode automatically (since children don't expect to be disoriented) and later call upon to reestablish their position. As a consequence, a rich array of experiments has investigated children's reorientation at diverse ages and in diverse environments. Humans reorient by the shape of a rectangular chamber as early as 12 months of age, and they continue to do so as adults (Hermer & Spelke, 1996). By 24 months, children reorient by the shape of both large and small rectangular environments (Learmonth, Newcombe & Huttenlocher, 2001), both when the surrounding walls are homogeneous and when they are distinctive in color or are furnished by distinctive landmarks (Hermer & Spelke, 1996; Learmonth, *et al.*, 2001). Children also reorient by the distances and directions of surrounding walls when they are tested in rooms of other shapes including

isosceles triangles (Lourenco & Huttenlocher, 2006; see Figure 3b, p. 247), and squares with distinctive protrusions (Wang *et al.*, 1999; see Figure 3c, p. 247). Nevertheless, children's reorientation shows limits that provide clues to the nature of the representations that guide them, and that allow investigators to track these representations across species, across cultures, and into the brain.

One signature limit of this core geometry system concerns the kind of layout information that it accepts: children reorient by the distances and directions of extended surfaces but not by the distances and directions of freestanding objects, even large ones. A recent series of experiments illustrates this limit (Lee & Spelke, 2010; see also Lew *et al.*, 2006; Wang *et al.*, 1999). Children were disoriented in a cylindrical environment with two large, stable columns that contrasted with the walls of the cylinder in brightness and color, positioned so that they both stood on one side of the room, separated by 90 degrees (Figure 4a, p. 247). When the columns stood flush against the cylindrical wall of the room, children used them to reorient themselves and locate a hidden object, both when that object was hidden directly at one of the columns and when it was hidden elsewhere. When the columns were offset slightly from the walls, however, children failed to use their positions to locate the hidden object. This failure did not stem from a failure to attend to or remember the relation of the object to the columns: if the object was hidden directly at one of the two columns, children searched only at the columns, showing that they appreciated their relevance to the task and used each column as a 'beacon', signaling a place where an object could be hidden. Children failed, however, to confine their search to the column with the correct directional relationship to the child: if the object was hidden at the column on the *right*, children searched equally at the two columns on the right and left. Columns only specified the child's position when they were placed flush against the walls, and therefore contributed to the shape of the surrounding surface layout.

Further experiments within this series revealed a second signature limit of children's geometry-guided navigation: children reorient by the distances and directions of extended 3D surfaces but not by the distances and directions of extended 2D patterns (Lee & Spelke, 2010; see also Lee, Shusterman & Spelke, 2006; Gouteux & Spelke, 2001; Wang *et al.*, 1999). Children were tested in the same cylindrical environment, but instead of viewing two 3D columns against the wall, they were presented with two 2D patches of the same angular size as the columns, made of the same material and contrasting dramatically from the surrounding walls in brightness, texture and color (see Figure 4b, p. 247). When an object was hidden at one of these patches, children confined their search to the two patches, showing again that they

detected them and used them as beacons to directly mark an object's potential location, but they failed to distinguish between them and therefore searched equally at the correct patch (e.g., the patch on the right) and the incorrect patch (on the left). Although the patches were clearly detectable, they did not alter the shape of the cylindrical environment so as to break its symmetry. Accordingly, they were not used by the geometric navigation system to specify the position of the child and the hidden object.

The blindness of this system of core geometry to 2D patterns is very striking and has led to some surprising findings. The most dramatic of these are findings from a series of studies by Huttenlocher and Lourenco (2007; Lourenco *et al.*, 2009). In these studies, children were disoriented in a square room, such that no information from the room's shape distinguished its four corners. In different conditions, the opposite pairs of walls of this room were distinguished from one another by 2D pattern information: for example, walls were covered with crosses vs. discs (see Figure 5, p. 248). Although the distinction between a cross and a disc is purely geometric, disoriented children disregarded this information and searched the four corners equally. Children disregarded distinctive surface markings in a variety of settings, including square rooms whose walls differed in color (alternating walls were red or blue) or in the presence vs. absence of patterning (alternating walls displayed black discs on a white background or were homogeneously gray). Children's failures did not stem from a failure to detect or use the pattern information, however, because children confined their search to the two directionally consistent corners in a final condition, in which opposite walls were covered with discs that differed in size and density (large, widely spaced discs on one pair of walls and small, narrowly spaced discs on the other pair).

Why did children succeed in this last condition, given that they failed to distinguish a wall with large discs from a wall with *no* discs? Recent experiments have addressed this question and reveal the exquisite sensitivity of this core geometry system (Lee, Winkler-Rhoades & Spelke, unpublished). When perceivers view equally distant surfaces covered by forms of identical shape but different sizes and densities, they perceive the surface with larger, less dense shapes as closer to them. This depth cue of *relative size* affects depth perception in infants as young as 7 months of age (Yonas, Granrud & Pettersen, 1985). Although the room in Huttenlocher & Lourenco's studies was square, the relative size cue led children to perceive it as slightly elongated, such that the walls with large circles appeared closer to them than the walls with small circles. This conclusion comes from experiments that tested two predictions from the thesis that relative size functioned as a depth cue. First, children should reorient in uniformly colored



environments even when they are only very slightly rectangular (because the effect of the relative size depth cue is subtle). Second, the relative size cue should interact predictably with other cues to depth to enhance or diminish children's reorientation.

To test these predictions, we first tested children in two uniformly colored rooms that were almost but not quite square, with walls varying in length in a ratio of 8:9 in one condition and 23:24 in the other. Children failed to reorient by the rectangular shape in the latter condition but succeeded in the former condition, providing evidence that quite subtle departures from a square shape were sufficient to guide reorientation. Next, we created three environments with circular patterns matching those used by Huttenlocher and Lourenco (2007): a square room, a subtly rectangular room such that the larger circles appeared on the closer walls, and a subtly rectangular room in which the larger circles appeared on the more distant walls. Children searched effectively in the first two conditions but not in the third. These findings provide evidence that the patterns varying in size and density acted as a depth cue, breaking the symmetry of the square room. As in all the previous experiments, 2D geometric patterns did not serve as independent information guiding children's reorientation. These experiments provide evidence that the core geometric system for navigation is guided by extremely subtle perturbations of 3D geometric structure. Additional studies bolster this conclusion by showing that children reorient effectively in environments where the only distinctive 3D shape is provided by a tiny rectangular frame or bump on the floor (Lee & Spelke, 2011; see Figure 6, p. 249).

A final signature property of the system guiding children's reorientation concerns its imperviousness to task manipulations that influence children's state of attention. Children's ability to use a distinctively colored wall, a 2D pattern, or a freestanding object as a beacon marking the location of a hidden object is strongly modulated both by children's attention to that feature and by their understanding of its potential relevance. For example, children who are told that a colored wall will help them find a hidden object subsequently use the wall to guide their search for the object, whereas children who are simply told that the wall has a pretty color do not (Shusterman *et al.*, in press). In contrast, children's use of the geometric configuration of an arrangement of walls is unaffected by these or other manipulations of attention. As children explore an environment, they automatically encode and remember their position within the extended 3D surface layout.

In summary, research on navigating children provides evidence for a system of representation that is sensitive to the geometric configuration of the extended surfaces within the navigable layout, but not to similar configu-



rations of movable objects, or two-dimensional surface markings (unless they create an illusory 3D surface pattern). This sensitivity, moreover, is independent of the child's state of attention or awareness of the future utility of this geometric information. These signature limits allow investigators to test for the same system in other animals, in human adults in diverse cultures and circumstances, and in specific systems in the brain. I will briefly mention each of these efforts.

Studies of reorientation began with research on rats, when Cheng (1986) and Gallistel (1990) developed the reorientation task. Reorientation by the shape of the environment has now been shown in a wide range of non-human animals, from primates to birds and even to ants (Weustrach & Beugnon, 2009; see Cheng & Newcombe, 2005, for a review). Untrained animals of all these species show the same signatures of reorientation found in children. Like children, for example, ants use 2D geometric patterns as beacons but only use the shape of the 3D environment for reorientation (Wystrach & Beugnon, 2009). Moreover, newly hatched chicks, like children, use both 2D patterns and large freestanding objects as beacons but fail to reorient by them (Lee, Spelke & Vallortigara, unpublished). Chicks also reorient by the same patterns of subtle geometric information as children, and mice show the same reorientation performance as children in square rooms containing patterning information evoking the relative size depth cue (Twyman *et al.*, 2009). Finally, training regimes that alter animals' attention to environmental features and awareness of their significance markedly change animals' navigation performance in relation to surface brightness, 2D patterns, and freestanding objects (e.g., Wystrach & Beugnon, 2009; Pearce *et al.*, 2001). These manipulations, however, have little or no effect on animals' response to surface layout geometry.

Research on human adults initially appears to contrast with these patterns: adults who are disoriented in a room with distinctive landmarks will use any and all landmarks to locate hidden objects: a finding to which I will return in the last section of this chapter. Beneath adults' much more successful performance, however, is the same system of geometry-guided navigation found in children and animals. For example, adults who participate in a reorientation experiment while engaged in a continuous verbal task show the signatures of children's and animals' search performance: they use objects and 2D patterns as beacons but reorient only by 3D surface layout geometry (Hermer-Vazquez *et al.*, 1999), unless task instructions specifically alert them to the task of locating landmarks (Ratliff & Newcombe, 2008). Even more dramatically, adults who are tested with no simultaneous interference, but who are deaf and have only limited access to a conventional sign language, reorient by the

shape of the layout as well as their hearing or linguistically more capable deaf peers, but they are less able to use a colored-surface landmark in a reorientation task (Pyers *et al.*, 2010). These findings suggest that a phylogenetically and ontogenetically ancient system of reorientation persists through human childhood and continues to function in adults, although adults use other cognitive resources, including those provided by their language, to compensate for its limitations. I return to the effects of language later in this chapter.

In research using neurophysiological methods, the same signatures have been found in the brains of navigating animals, in areas whose activity specifies the animal's location ('place cells'), heading ('head-direction cells'), or motion ('grid cells'). Place cells in the hippocampus, and head-direction and grid cells in nearby regions of the cerebral cortex, discharge in patterns that are systematically affected by the animal's distance and direction from the 3D extended surfaces within the chamber (O'Keefe & Burgess, 1996; Lever *et al.*, 2002; Solstad *et al.*, 2008). In contrast, the activity of these cells is unaffected by the positions of freestanding objects or the colors and textures of the chamber's walls (Lever *et al.*, 2002; Solstad *et al.*, 2008). Interestingly, place cell activity also changes with experience, in patterns that suggest that objects, surface textures, and other environmental features come to be encoded as animals learn their relevance (Lever *et al.*, 2002). Studies of very young rats provide evidence that this navigation system is in place very early in development. The activity of place cells and head-direction cells is detectable, in infant rats, as soon as the rats begin to locomote; the activity of grid cells develops shortly thereafter (Wills *et al.*, 2010). All these findings parallel the findings from behavioral studies of young children.

Recent research using methods of functional brain imaging provides evidence for place and grid cells in human adults as well, consistent with the behavioral research described above (Doeller *et al.*, 2008, 2010). The human hippocampus is activated when adults learn to identify locations in a virtual environment in relation to extended 3D surfaces (Doeller & Burgess, 2008; Doeller *et al.*, 2008). In contrast, when adults learn to identify locations in relation to freestanding landmark objects, a different brain region in the striatum, is associated with their task performance. Hippocampal activity associated with learning an environmental location in relation to an extended surface in the virtual layout is markedly impervious to effects of attention and interference; in contrast, activity in the striatum, associated with learning of landmarks, shows marked effects of attention. These studies reveal a remarkable convergence across humans and rodents, and across behavioral and neurophysiological methods, in the core mechanisms for encoding the shape of the surrounding surface layout.

Finally, an exciting new line of research hints that the core system of geometry may have a specific genetic basis. The research focuses on adults with Williams Syndrome, a developmental disability stemming from a genetic deletion that produces a variety of structural and cognitive abnormalities, including impairments in spatial reasoning. Lakusta, Dessalegn & Landau (2010) focused on the latter impairments and found that adults with Williams Syndrome performed a wide variety of spatial tasks at roughly the level of three-year-old children. When they tested the reorientation performance of adults with Williams Syndrome in a homogeneously colored, rectangular room, however, they discovered a novel pattern of performance: in contrast to all the findings reviewed above, adults with Williams Syndrome searched equally at the four corners of the rectangular room. Their performance did not stem from a failure to remember the location of the hidden object, because they performed well when tested in the same room after a delay of equal duration but without disorientation. Their performance also did not stem from any debilitating effects of the disorientation procedure, because they performed fairly well when tested, after disorientation, in a rectangular room with one distinctively colored wall. Tests of typical 3-year-old children revealed a striking double dissociation: whereas children successfully navigated in accord with the shape of the environment and failed to navigate in accord with the colored wall, adults with Williams Syndrome did the reverse. This developmental disability therefore seems to produce a specific deficit in the core system for navigating by layout geometry.

At the time of writing of this chapter, the genetic and epigenetic processes that sculpt this core system still are unknown, but they are now open to study. Williams Syndrome is caused by a known genetic deletion, and mouse models both of this syndrome and of reorientation now exist. Future experiments on mice therefore can probe the nature of the developmental mechanisms by which this ancient system of geometric representation emerges or goes awry.

In summary, studies of navigation across human ontogeny, across vertebrate and invertebrate phylogeny, across human cultures and languages, and across levels of analysis from cognition to neurons to genes, all provide evidence for a navigation system that relies on the geometry of the surrounding surface layout. But I end this section with a crucial question: What kinds of geometric information about the surface layout does this system represent? The last navigation research that I will describe provides evidence that children, adults and animals maintain and reestablish their sense of place by representing with two fundamental properties of Euclidean geometry – *distance* and *direction*. However, navigating children, adults and animals do not

represent the two most important geometric properties that characterize the shapes of visual forms: *length* and *angle*.

Children's insensitivity to the angles at which walls meet at corners was first shown by Hupbach & Nadel (2005), who tested children in a rhomboid environment consisting of four walls that were equal in length but met at unequal angles. In a recent replication and extension of this research, we found that such children are strikingly insensitive to angle, even in the simplest environments (Lee, Sovrano & Spelke, in review). These tests presented 2- to 3-year-old children with fragmented rhomboid arrays within a uniform cylindrical environment (see Figure 7a, p. 250). In one condition, we removed the distinctive angle information available in a complete rhombus by presenting four walls in a rhomboid arrangement, separated by gaps where the corners had been. When a toy was hidden within this array and children were disoriented, they confined their search to the two geometrically appropriate locations, providing evidence that children can reorient in fragmented as well as continuous environments and that they do not need to see corner angles directly in order to locate themselves in relation to extended surfaces. In a second condition, we removed these surfaces and presented just the four corners of the rhombus, positioned so that they were equidistant from the room's center. Although the pairs of opposite corners presented markedly different angles (obtuse angles of 120 degrees and acute angles of 60 degrees), disoriented children were not guided by this distinction and divided their search equally among the four hiding locations. Children's reorientation evidently is not guided by the angles at which surfaces meet.

Two further experiments in this series presented children with fragmented rectangular arrays and tested whether children reorient in accord with the lengths of surfaces (see Figure 7b, p. 250). First we removed the distinctive length information that is present in any complete rectangle by presenting four surfaces of equal length, positioned so as to form a rectangular array whose major and minor axes differed by a 2:1 ratio. When a toy was hidden within this array, disoriented children confined their search to the two geometrically appropriate locations, providing evidence that they do not need to see surfaces of different lengths in order to position themselves within a rectangular space. Then we presented four surfaces at two different lengths, positioned so as to form a square enclosure. Although one pair of surfaces was twice as long as the other pair, disoriented children searched the four locations at random, without regard to the length distinctions.

These new findings suggest that it is misleading to characterize the geometry-guided reorientation system as sensitive to the *shape* of the surrounding layout (as I and others have done: e.g., Hermer & Spelke, 1996).

We will see in the next section that a fundamental property of shape representations is their invariance over transformations of scale, and their dependence on the angle and length relations between the parts of an object or the contours of a form. Children, however, do not respond to these angle and length relations, and neither do the rats tested in neurophysiological experiments. When a square room is quadrupled in area, place cell activity remains anchored to the absolute distances and directions of one or more walls; place fields do not move and expand to preserve their relation to the environment's global shape (O'Keefe & Burgess, 1996).

In summary, these findings provide evidence for a core system of geometric representation guiding navigation. The system, however, has two general sorts of limits. First, it applies only to the extended surfaces in 3D navigable layouts, not to 2D patterns or freestanding objects. Second, it represents the navigator's position relative to the absolute distances and directions of those surfaces, but it does not represent the geometric relationships of angle and length that each of those surfaces bears to the other. In the next section, I describe a second core system of geometric representation that has neither of these limits.

### **Geometry for object recognition**

Although animals and young children are strikingly oblivious to surface markings and relations of length and angle in the large-scale navigable layout, they are highly sensitive to these geometric relationships when they appear in small pictures or objects. This conclusion is supported by many decades of experiments on form perception and object recognition in animals and children (see Gibson, 1969, for a classic review of the earlier literature). It also is supported by research on human infants who cannot yet locomote independently (and hence cannot be tested in the above navigation tasks). I begin with the latter findings.

Decades of experiments have investigated sensitivity to angle and length in human infants, sometimes as early as a few hours after birth (Slater, Mattock, Brown, & Bremner, 1991). In one set of studies, infants were habituated to two lines crossing at a constant angle, presented at a number of different orientations. Then they were tested with new displays presenting the same shape in a previously unseen orientation, or a shape that presented the same two lines, joined at a different angle. Infants generalized habituation to the former displays and looked longer at the latter ones, suggesting that they are sensitive to angle (Schwartz & Day, 1979; Slater *et al.*, 1991). Similar experiments showed that infants also are sensitive to length (Newcombe, Huttenlocher & Learmonth, 1999).

Further studies suggest two further differences between infants' perception of the shapes of visual forms and children's and animals' use of geometry in navigation. First, research on animals provides evidence that navigation depends on an encoding of the absolute, not relative, distances of extended surfaces: positions within a space are not encoded in a scale-invariant manner (O'Keefe & Burgess, 1996). In contrast, infants' encoding of the shapes of visual forms is largely scale-invariant. In one series of studies (Schwartz & Day, 1979), infants were habituated to a small form and then were tested with form that changed either in one dimension (altering their shape and size) or in two dimensions (preserving their shape but creating a greater change in size). In contrast to the rats in the navigation tasks described above (O'Keefe & Burgess, 1996), infants showed greater dishabituation to the change in shape, providing evidence that they perceived the distinctive lengths of the contours of a form in a relative, not absolute, manner.

Second, navigating animals and young children encode the directional relations that distinguish an array from its mirror image. In particular, children readily distinguish the corner of a rectangular room where an object is hidden from its mirror image: indeed, only this relationship distinguishes the two geometrically congruent corners of such a room from the other, incongruent corners. In contrast, studies of form perception in infancy, probing infants' detection of the directional relations that distinguish a pattern from its mirror image, have yielded mixed findings. In most studies, infants fail to distinguish between a 2D geometrical form and its mirror image (Lourenco & Huttenlocher, 2008). In the rare studies where they succeed, infants are shown a succession of events that may induce a process of 'mental rotation', and only subsets of infants succeed in engaging this process (Quinn & Liben, 2008, Moore & Johnson, 2008). These findings provide evidence that visual forms are represented in a manner that is largely blind to sense distinctions.

These properties of visual form representations are largely invariant over human development, across different human cultures, and across animal species. I will describe just one set of recent experiments probing the later development of sensitivity to length, angle and direction in humans (Izard & Spelke, 2009; Izard *et al.*, 2011b). The experiments, conducted on adults and on children ranging from 4 to 10 years of age in both the U.S. and in a remote region of the Amazon, revealed a developmentally invariant pattern of performance that agreed closely with the findings from studies of form perception in infants. At all ages, children and adults detected angle and length relationships with relative ease, but they failed to detect directional relationships until adolescence.

The experiment used a deviant detection paradigm (after Dehaene *et al.*, 2006) in which five forms in the shape of an L shared a geometric property that a sixth L-shaped form lacked. Participants were presented with the six forms in a random arrangement and at random orientations, and their task was to detect the geometrical deviant. On different trials, the deviant form differed from the other forms in line length, angle, or sense (see Figure 8, p. 251). On pure trials, all the forms were otherwise identical; on interference trials, the forms varied along a second irrelevant dimension. A comparison of the latter trials to the pure trials served to assess whether length, angle or sense was processed automatically and interfered with detection of the relevant dimension.

There were two main findings. First, participants of all ages, and in both cultures, showed highest sensitivity to angular and length relations and lowest sensitivity to the sense relation that distinguishes a form from its mirror image. Second, variations in angle and length interfered with one another and with processing of sense, but variations in sense had no effect on processing of angle or length. At all ages, therefore, sense is difficult to detect and easy to ignore.

As a large literature documents, adults' ability to distinguish a form or 3D object from its mirror image often requires the application of a mental rotation to align the objects (Cooper & Shepard, 1973). After rotation, two visual forms or objects can be compared directly by a process of template matching, with no need to represent their abstract sense relations. Mental rotation therefore appears as a strategy that is applied to compensate for the absence of abstract, orientation-independent representations of sense. In contrast, the deviant detection study showed that angle and length relations are processed reliably and rapidly in figures that vary in orientation, as well as figures that vary in sense. Processing of angle and length occurs very readily, in arrays that differ in a variety of other geometric properties including orientation and sense.

The findings of studies of 2D form perception complement those of a large body of research on 3D object recognition. Objects are recognized primarily on the basis of their shapes, beginning in early childhood (Smith *et al.*, 2002; Smith, 2009) and continuing through adulthood, although the coordinate systems within which shapes are represented is a matter of persisting dispute (see Biederman & Cooper, 2009; Riesenhuber & Poggio, 2000). Adults across cultures recognize meaningful objects by detecting basic Euclidean properties such as the presence of straight edges and parallel surfaces (Biederman, Yue & Davidoff, 2009). When objects are depicted in line drawings, junctions where distinct lines meet provide particularly important information that adults and children use to recognize them (Biederman, 1987;



Figure 9). In both behavioral and neuroimaging studies, moreover, shape-based object recognition has been found to be invariant both over a wide range of scales and over reflection (Biederman & Cooper, 2009), providing evidence for sensitivity to differences in length and angular relationships but not to the directional relationship that distinguishes a shape from its mirror image. Finally, variations in object shape elicit activation in specific regions of the occipital and temporal cortex of the brain (Grill-Spector, Golarai, & Gabrieli, 2008; Reddy & Kanwisher, 2006). These regions respond to the shapes both of three-dimensional objects and of two-dimensional forms, both in humans (e.g., Kourtzi & Kanwisher, 2001) and in non-human primates (e.g., Kriegeskorte *et al.*, 2008), further suggesting that common cognitive mechanisms underlie perception of the shapes of two-dimensional visual forms and of three-dimensional manipulable objects.

In summary, research provides evidence for a core system for representing the shapes of movable, manipulable objects. The system endures over human development (Izard & Spelke, 2009) and appears to be culturally universal (Dehaene *et al.*, 2006; Izard *et al.*, 2011b). It represents object shapes over variations in orientation, size, and sense, so as to highlight two fundamental properties of Euclidean geometry, *length* and *angle*. Yet this system falls short of full Euclidean geometry. It fails to apply to the large-scale, navigable layout, as evidenced by children's lack of response to angular relations in a rhombic room or to length relations in a rectangular room (Lee *et al.*, in review). Moreover, it captures Euclidean *distance* and *angle* but not *sense*, and therefore fails to distinguish a form from its mirror image.

The system by which children and adults analyze the shapes of objects so as to recognize them, contrasts markedly from the system by which children and adults analyze the geometric properties of the surface layout so as to maintain or recover their sense of orientation. Representations of visual shapes apply primarily to 2D visual forms and to manipulable, movable objects; representations of the shape of the navigable layout specifically exclude these 2D forms and objects. Moreover, representations of object shapes are highly sensitive to relative length and angle and generalize over changes in distance (and object size) and direction (and object sense relations). Representations of places in the layout show the reverse pattern: high sensitivity to distance and direction and little or no sensitivity to length and angle. Finally, representations of object shape depend primarily on corners and other junctions where objects meet: a drawing of an object is still recognizable when portions of its smooth lines are deleted, but it suffers with the deletions of corners and intersections (Figure 9). Representations of the navigable layout again show the reverse pattern: they are preserved over deletion



**Figure 9.** Adults and children can recognize common objects based on 2D drawings of their shapes alone (left). When contours are deleted from the centers of the lines or curves that indicate object parts with planar or smoothly curved surfaces (center), objects continue to be recognized. In contrast, recognition suffers when the same amount of contour is deleted from regions where 2 or more lines meet at a corner (right; reprinted from Biederman, 1987).

of corners and perturbed by interruption of smooth, continuous surfaces (Figure 7, p. 250). The two systems of geometry found in young children therefore differ in striking ways. The system of formal Euclidean geometry that is most intuitive to adults, in contrast, bridges these differences.

### **Beyond core knowledge**

The contrasting properties of the two core systems of geometry suggest that more powerful and abstract system of geometrical knowledge could arise if the representations from the core systems could be productively combined. If children could systematically combine the geometric properties they extract from large-scale navigable layouts and from small-scale forms, they might overcome the limits on the domains of application of these systems and increase the power of their geometrical analyses. By combining these systems, children might navigate by angle and relative length as well as by direction and absolute distance. Moreover, children might distinguish forms and objects from their mirror images by viewing those forms and objects through the lens of geometry-guided navigation, using real or mental rotation to view objects from changing perspectives.

By productively combining their systems of navigation and visual form analysis, children might also develop conceptions of truly abstract geometrical objects. Consider one such object: a line with unbounded extent and no thickness. Such lines do not exist in representations of navigable layouts, whose geometric representation consist only of extended surfaces at particular distances and directions. Such lines also do not exist in representations of 2D forms, because every visible surface marking has some thickness. Consider, however, the concepts that could emerge when children begin to use arrays of 2D forms as *pictures* or *maps* that depict a 3D surface layout. In a picture of a scene, thin markings (that we call ‘lines’, despite their visible thickness) depict the boundary between one object or surface and another. In the scene itself, however, that boundary extends only in one dimension: it has no thickness. In this sense, there are no visible ‘lines’ in the 3D scene, but only edges where a surface ends. When a child comes to view a marking on a two-dimensional surface as representing an edge in a 3D layout, however, he or she may come to grasp the more abstract geometrical object that is common to these two arrays: a line that extends only in one dimension.

To date, many questions remain concerning the development of children’s understanding of maps, pictures, and the abstract geometrical objects that they use to connect these representations with representations of 3D navigable layouts. I hypothesize that the system of abstract geometrical intuitions with which I began this paper – the system that is constructed by

children and shared by adults in diverse cultures – is constructed by processes that productively combine the two kinds of core geometrical representations found in animals and young children. Visual symbol systems like maps may provide one medium within which these two kinds of representations are productively combined. I end by discussing research that focuses on a different medium that children use to effect this productive combination. In the experiments that I will discuss, children do not use visual symbols to combine representations of places and objects, but a different representational system that is explicitly combinatorial: language.

These experiments return us to the reorientation task, and to the finding that young children reorient in accord with the distances of surfaces but not their colors. In a new series of studies, children's reorientation was tested in a rectangular chamber with three white walls and a fourth wall of a distinctive color. When this environment is large and children are disoriented, they use the large colored wall to distinguish between the two geometrically correct corners (Learmonth *et al.*, 2001), especially when the object is hidden near the colored wall (Shusterman *et al.*, in press), suggesting that the size of the wall draws children's attention to it and engages a process of beacon guidance (Lee & Spelke, 2010). When the environment is small, however, children reorient only by the geometry of the room, confining their search to the two corners with the correct distance and directional properties, (Hermer & Spelke, 1996). Although young children can use wall colors to specify beacons, and wall lengths and directions to specify their own position and heading, they do not combine these sources of information.

Children's behavior changes, however, at the time when they begin to master spatial expressions involving the terms *left* and *right*: about 6 years of age. Interestingly, the development of this ability correlates with the acquisition of spatial language in individual children (Hermer-Vazquez *et al.*, 2001) and it is accelerated by teaching children the terms *left* and *right* (Shusterman & Spelke, 2005). But what role does language play? Does language training serve only to increase children's attention to the colored landmark (as in the studies by Shusterman *et al.*, in press, already described)? Alternatively, does language serve as a more productive medium for combining information about the spatial layout with information about landmark objects?

Recent studies of adult speakers of Nicaraguan Sign Language (NSL) serve to address this question (Pyers, Shusterman, Senghas, Spelke & Emmory, 2010). NSL began to emerge in the 1970s among children attending a new school for the deaf, and it is the primary language of the school's graduates. The language developed consistent grammatical structures, however, only over successive generations of students. The first wave of students converged

on a common language that includes nouns and verbs but lacks many of the grammatical devices of fully developed signed or spoken languages. These 'first-cohort' speakers have no consistent means for expressing or interpreting spatial relationships such as *left of X*. Over the course of a single conversation, they may shift from conveying left/right relations from their own perspective or from the perspective of their conversational partner (Senghas *et al.*, 2004). Later generations of students entered the school after the development of the language was more advanced, and their language is correspondingly richer and more systematic. In particular, 'second-cohort' speakers are more consistent in their use of expressions for left-right relationships, and they communicate these relationships more effectively (Senghas *et al.*, 2004). Except for these language differences, members of different generations are similar: all the members of the first and second cohorts are now adults, and most of them live in the same city. Studies of these two cohorts therefore allow investigators to test whether differences in their language relate to any differences in their performance on non-linguistic spatial tasks.

To address this question, Pyers *et al.* (2010) presented first- and second-cohort NSL speakers with the task of reorienting in a rectangular environment with a single distinctively colored wall. After completing both this task and a second spatial task, participants were asked to describe a variety of spatial arrays, and their signed expressions were analyzed and compared across cohorts. As expected, second-cohort NSL signers showed superior language skills on a number of measures, including two measures of spatial language: they maintained a more consistent coordinate system when using expressions for *left* and *right*, and they were more consistent in their placements, within the signing space, of signs for landmark objects.

The most interesting findings come from comparisons of these adults' performance on the reorientation task. First, although signers in both cohorts confined their search for the hidden object to the two geometrically appropriate locations, those in the second cohort were better able to distinguish between those locations on the basis of the colored wall. Second, across the entire sample, use of the colored wall for reorientation correlated with one aspect of spatial language: the consistency of signing of expressions involving the relations *left* and *right*. Importantly, performance in the reorientation task did not correlate with other differences in language proficiency, and proficiency at left/right spatial language did not correlate with performance on the other spatial task. Thus, these findings do not reflect individual differences in the overall proficiency of language or spatial cognition, but a specific effect of spatial language on performance on the overtly nonverbal, reorientation task.

The NSL speakers in the studies of Pyers *et al.* (2010) all had developed some degree of spatial language. What is navigation like in a human adult with normal nonlinguistic cognitive abilities but no spatial language at all? A suggestive answer to this question comes from a recent case study of an adolescent referred to as IC (Hyde *et al.*, 2011), who communicated with his family by means of an idiosyncratic gestural system, or *homesign* (Goldin-Meadow, 2003). IC had little or no formal education or conventional language, but is he highly intelligent and skilled both at navigation and at numerical reasoning. In one series of studies, IC was asked to describe in gestures a set of images of objects under conditions designed to elicit object names (e.g., monkey in one image, a tree in another), number words (e.g., two monkeys in one image, three monkeys in another), or spatial expressions (e.g., a monkey above a tree in one image and below a tree in the other: examples of these images appear in Figure 10, p. 251). IC spontaneously and readily produced gestures designating kinds of objects and numbers, but he never produced gestures designating the spatial relationships among objects. (The only apparent exception occurred in the case of the relations *on* and *under*, where IC's gestures suggested that he distinguished these images by referring to actions or mechanical relationships rather than spatial relations). Across three testing sessions, and despite numerous hints and attempts at teaching, IC never produced any expressions that distinguished arrays of two objects by their directional (left/right) relationships.

IC was, however, a superb navigator. He traveled around his home city with ease, and was reported to have done so from a very early age. Could IC combine spatial and geometric information so as to locate an object to the left or right of a distinctively colored wall? Our first attempts to address this question used the reorientation task, and they failed decisively: IC reoriented perfectly in every environment in which he was tested, including a circular room with no geometric or non-geometric structure whatsoever! Evidently, our best efforts to create a perfectly symmetrical environment were not good enough to fool IC, who reoriented himself by detecting extremely subtle asymmetries in our testing environment.

Our next attempts therefore tested IC's memory for movable spatial arrays. Under these conditions, IC reliably used the shape of the environmental configuration to specify the location of a hidden object, and he also reliably used the distinctive color of a landmark object as a direct beacon to the hidden object's location, consistent with past research on young children. Nevertheless, IC failed to combine these sources of information reliably. These data provide suggestive evidence that spatial language fosters this combinatorial capacity.

How might spatial expressions such as *left of the blue wall* serve to combine geometric and landmark information automatically and productively?

I suggest that this effect depends on three central properties of all natural languages (Spelke, 2003). First, languages consist in part of a lexicon of words for many kinds of entities, including words for environmental features (*wall*), their properties (*far, blue*), and their spatial relationships to other environmental features (*left, behind*). Second, languages consist in part of a set of rules for combining these words to form expressions, and those rules depend only on the grammatical properties of the words that they serve to combine, not on their content domains. Although *far* refers to property that the core navigation system can represent whereas *blue* does not, both are adjectives, and so any grammatical expression that includes one could instead include the other. Third, the meanings of the expressions of a language follow from the meanings of its words and the rules for combining them: If one learns a new object term (say, *iPad*) and already knows the meaning of expressions like *the left side of the wall*, one needs no further learning to know the meaning of new expressions such as *the left side of the iPad*.

With these three properties, language could serve as a medium in which information about object properties, and information about the shape of the surrounding layout, could be productively combined. With a cognitive system for representing objects, children can learn terms like *red* and *triangle* by mapping words and expressions to object representations. And with a separate cognitive system for representing distances and directions in the navigable environment, children can learn terms like *far* and *left* by mapping words and expressions to representations of the extended surface layout. The combinatorial machinery of natural language could do the rest, specifying the meanings of expressions that combine these terms, and thereby serving as a medium in which information from distinct cognitive systems can be productively integrated.

Language might, however, improve children's navigation in a different way. Perhaps language does not allow children to combine core representations but to bypass them. When children learn an expression like *left of the blue wall*, they may gain a new means for encoding properties of the environment that frees them from their core systems of geometry. Research on adults with Williams Syndrome, described earlier in this chapter, sheds light on this possibility (Lakusta *et al.*, 2010).

As noted, WS adults appear to lack altogether the core system of geometry for navigation: they show no ability to reorient themselves by representing the distances and directions of the walls of a rectangular room. In contrast, these adults have some spatial language and also some ability to use the distinctive color of a wall to specify the location of a hidden object. If language serves to bypass geometric representations, then these two abilities should be related to one another as they are for Nicaraguan signers:



WS adults with more consistent spatial language should be more consistent in their search to the left or right of a colored wall.

Lakusta, Dessalegn and Landau (2010) tested this prediction and decisively disconfirmed it. WS adults showed no relation between the consistency of their spatial language and the consistency of their reorientation performance in a room with one colored wall (Lakusta *et al.*, 2010). These findings support the hypothesis that language serves to combine core representations. Because adults with Williams Syndrome lack a core representation of layout geometry, their spatial language cannot play this role, however well it develops. Adults with WS likely outperform young children on navigating by a colored wall, because they have learned that they must attend to landmarks in order to maintain a sense of their own location. Like children and many animals, WS adults are able to attend and navigate by local landmarks, including color. Unlike other human adults, however, WS adults lack geometric layout representations and so cannot use language to combine landmarks and productively with geometric representations of the navigable layout.

### **Toward a developmental cognitive neuroscience of education**

The research described in this chapter suggests that our simplest, abstract geometrical intuitions have a complex history. They are rooted, first and foremost, in specialized neural systems that guide navigation and object perception both in animals and in humans from infancy onward. The representations delivered by these systems then are combined by a host of representational devices, including pictures and maps. Perhaps above all, they are combined productively as children master the words and rules of their natural language. Together, all these developments may underlie the universal abilities of adults, and of children from the age of 10 years, to navigate by purely geometric maps, to deduce the unseen position and size of the third angle of a triangle, and to intuit the behavior of points so small they have no size, and of lines that are infinitely thin, perfectly straight, and never end.

Can the insights from this research serve to foster children's education in mathematics? Because formal geometry is not explicitly taught in most elementary mathematics curricula, research linking children's early developing geometrical abilities to their later learning of geometry is only beginning. Even at this early stage, however, research on the cognitive and neural foundations of geometrical reasoning suggest ways that education in geometry might be enhanced for all students, including the youngest ones.

Formal geometry tends to be introduced in contexts that are far removed from the tasks of navigation and visual form analysis in which our geometrical intuitions may originate. The displays on which it focuses are tracings

created by a ruler and compass; the processes that it engages are those of logical reasoning, especially theorem-proving. Many students fail to be engaged by these tasks or appreciate their relevance to all the activities in which geometry is naturally engaged. In contrast, navigation and visual form analysis are tasks that young children enjoy, and that are both challenging and satisfying. Instruction in geometry might be both more enjoyable and more meaningful to students if it were introduced early in the educational system, in these task contexts.

Much of Euclidean geometry could be taught in the context of real or virtual tasks of navigation and form analysis, as in the studies of Shusterman *et al.* (2006) and Izard *et al.* (2011a) with which I began this chapter. Using these tasks, a mathematics curriculum could build upon the geometrical knowledge that young children already possess. Recent research bridging education and cognitive neuroscience suggests fruitful ways to enhance children's literacy by educational programs building on their preexisting representations of language, as the chapters in this volume by Dehaene and Kuhl describe. Instruction in geometry, so necessary for the development of higher, uniquely human cognitive skills, also may be enhanced by building on the foundational systems of geometrical analysis that arise in us as infants. These systems are embodied in distinct and early developing brain systems that have been intensively studied both in humans and in animals; many of the fundamental properties of these brain systems are now understood. Research that uses their findings to craft and test new educational initiatives should be a high priority for investigators in the fields of education and developmental cognitive neuroscience.

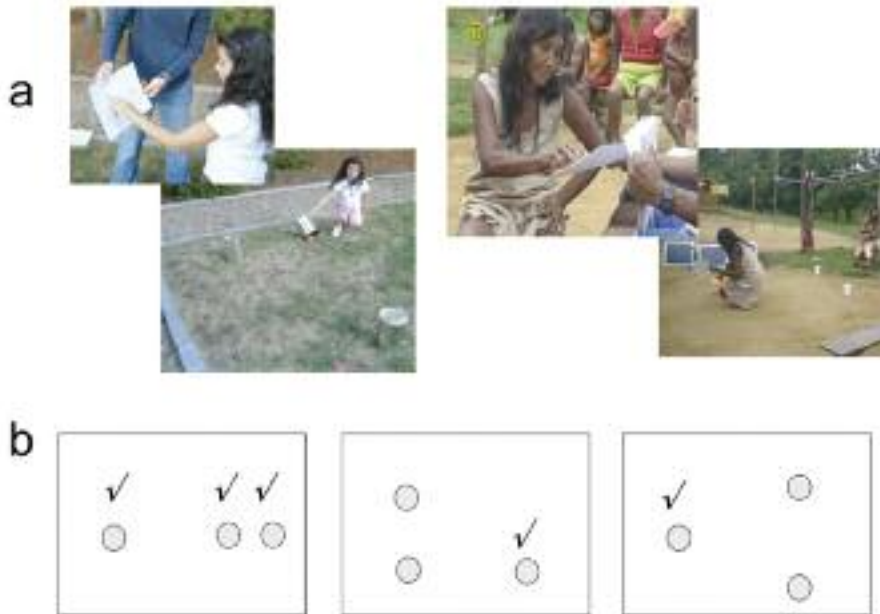
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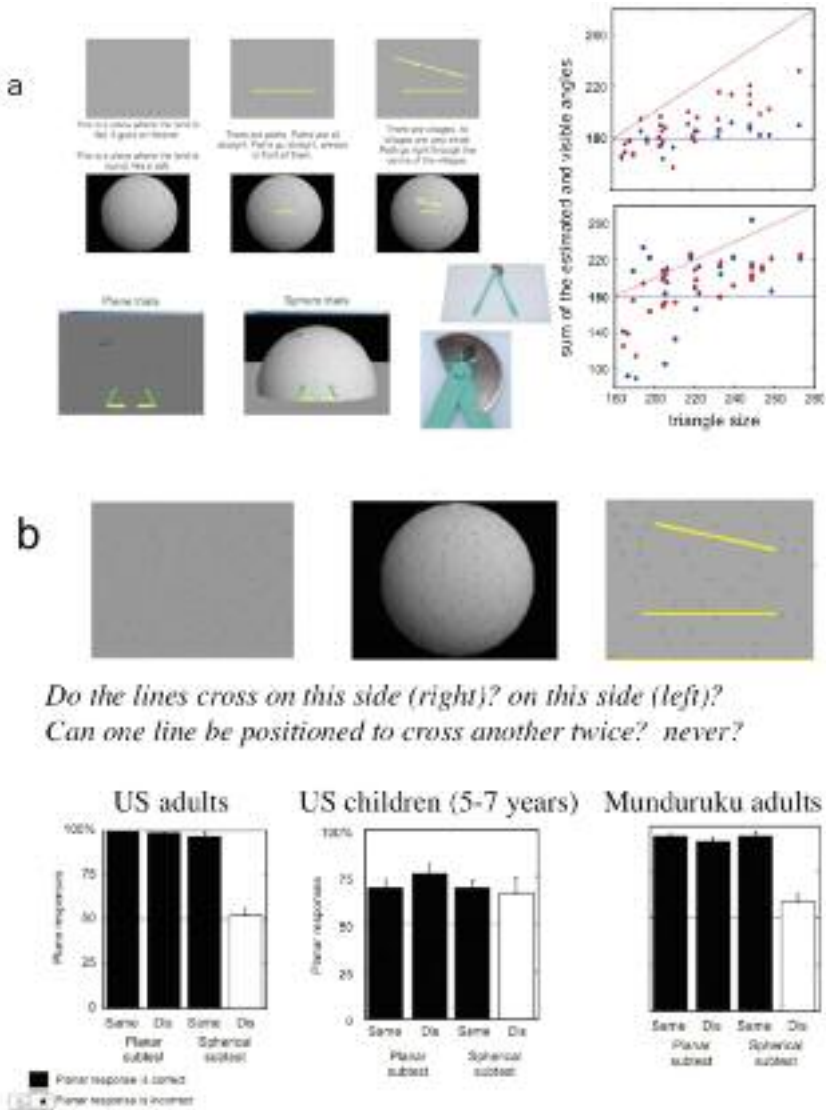
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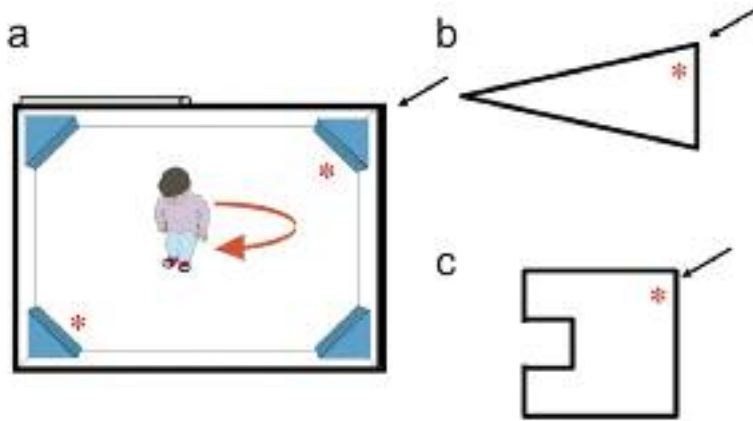


**Figure 1.** Displays and performance in a test of navigation by purely geometric maps in adults and children in two cultures. (a) The experimental settings and a sample task in the studies with U.S. children (left) and Amazonian adults (right). (b) The three map configurations used in the study with 4-year-old children; adults in the two cultures were tested with the triangular maps only. Arrows indicate the target locations at which 4-year-old children were successful; adults in both cultures performed above chance at all locations (after Shusterman *et al.*, 2008, and Dehaene *et al.*, 2006).

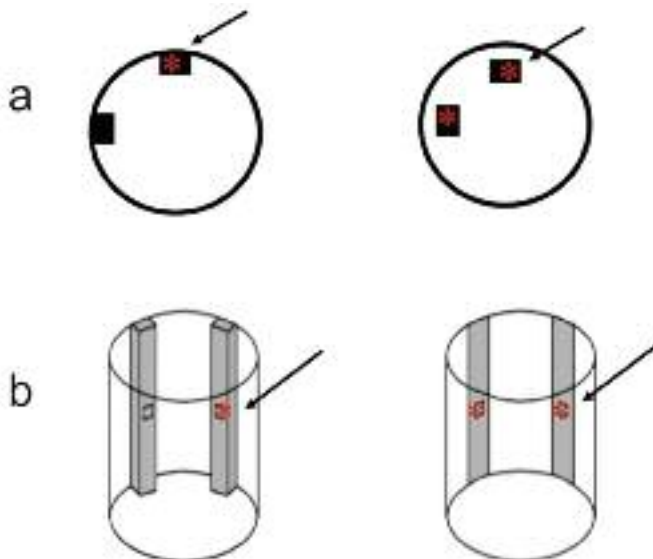


**Figure 2.** Displays for tests assessing abstract geometrical intuitions. (a) The displays and instructions for a study of triangle completion. Participants were introduced to a plane or sphere (top left) and then judged the position and angle at which the two lines met at the unseen apex of a triangle (below left). Scatterplots show the mean sums of angle estimates with the two visible angles on each trial, for U.S. adults (top right) and 6-year-old U.S. children (below right). Planar trials appear in blue and spherical trials in red; solid lines indicate correct responses. (b) Example displays and questions for a study of intuitions about points and lines. Participants gave verbal yes/no judgments to all questions. Percent planar responses are given separately for the plane and the sphere, and separately for judgments for which answers do and do not differ on the two surfaces (after Izard *et al.*, 2011a).





**Figure 3.** Overhead view of the arrays used in tests of children's reorientation in arenas with distinctive shapes. Arrows indicate the location of the hidden object (counterbalanced across children and rotated into alignment here to convey the findings); asterisks indicate the location(s) of children's search (a) in rectangular arenas (after Hermer & Spelke, 1996), (b) in isosceles arenas (after Lourenco & Huttenlocher, 2006), and (c) in a square arena with a symmetry-breaking bump in one wall (after Wang *et al.*, 1999).



**Figure 4.** Arrays used in tests of children's reorientation by geometric perturbations in the 3D extended surface layout, by geometrically similar configurations of freestanding objects, or by projectively similar configurations of 2D surface markings. Arrows indicate the location of the hidden object; asterisks indicate the location(s) of children's search. (a) Overhead view of arrays in which the same columns were flush against the boundary surface or freestanding. (b) Oblique view of arrays in which the columns were 3D projections or 2D patches (after Lee & Spelke, 2010).

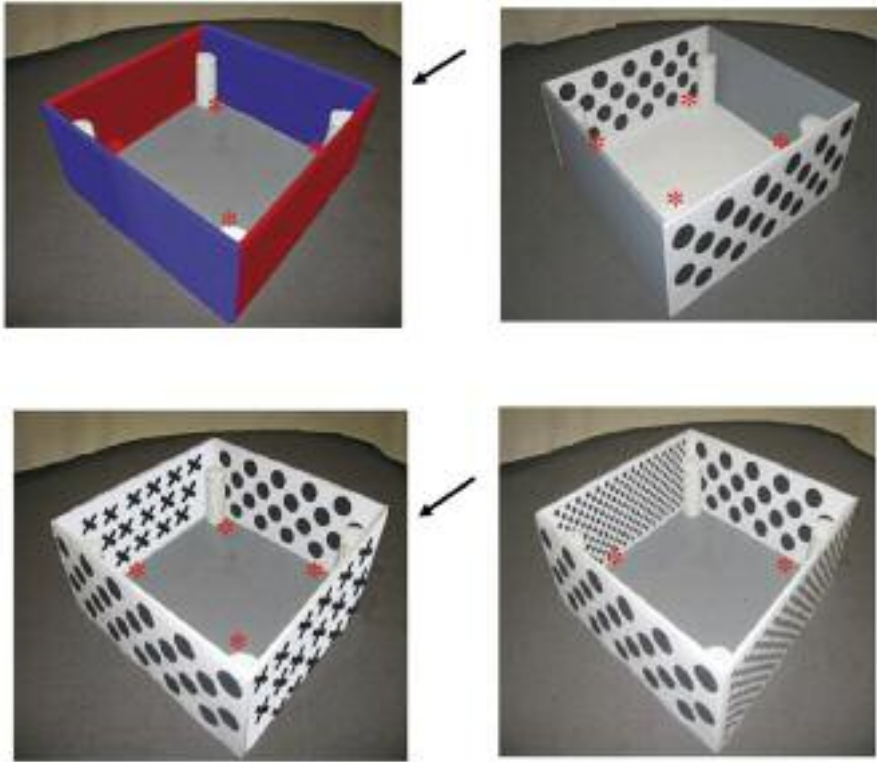
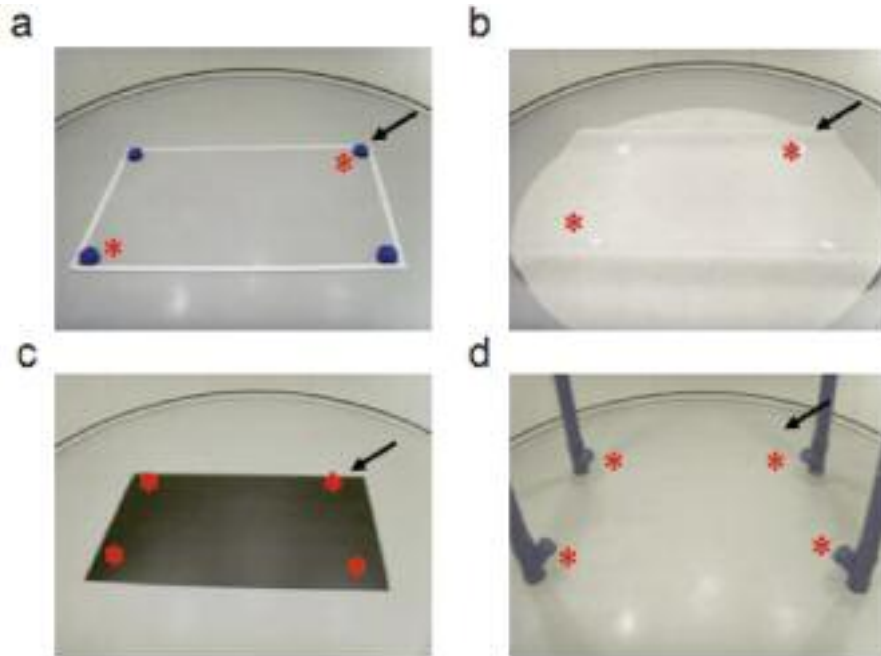
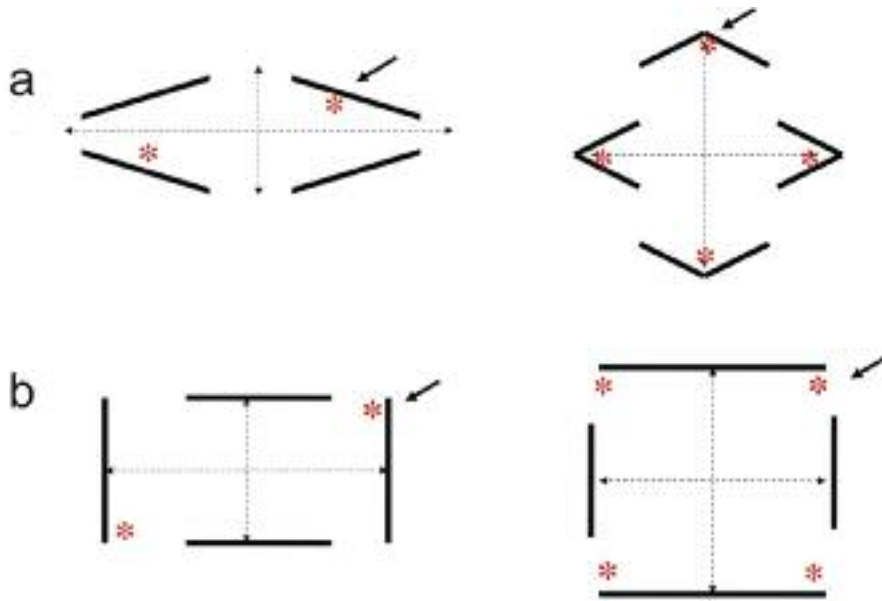


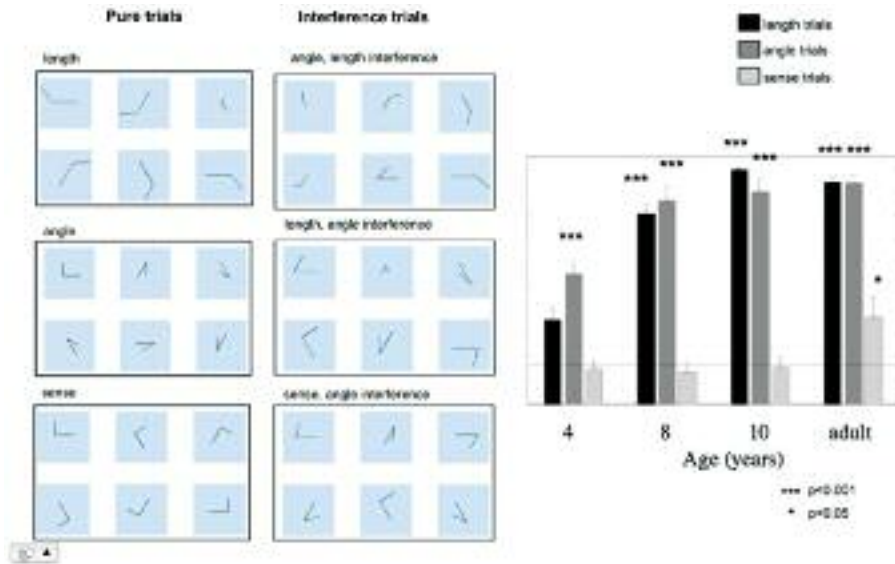
Figure 5. Arrays used in tests of children's reorientation in square arenas whose alternating walls differed in color, patterning, or relative size. Arrows indicate the location of the hidden object; asterisks indicate the locations of children's search (after Huttenlocher & Lourenco, 2007).



**Figure 6.** Arrays used in tests comparing children's reorientation by subtle perturbations in the 3D surface layout caused by (a) a 2-cm frame or (b) two gradual bumps, to their reorientation by prominent brightness edges in (c) 2D patterns or (d) freestanding objects. Arrows indicate the location of the hidden object; asterisks indicate the locations of children's search (after Lee & Spelke, 2011).



**Figure 7.** Arrays used in tests comparing children's abilities to reorient by angle and length to their abilities to reorient by surface distance. Dashed arrows indicate the distance relationships within the test spaces. Solid arrows indicate the location of the hidden object; asterisks indicate the locations of children's search in (a) fragmented rhomboid arrays and (b) fragmented rectangular arrays (after Lee *et al.*, in review).



**Figure 8.** Arrays used in tests assessing the sensitivity of children and adults to simple forms differing in length, angle, or sense. Arrays on the left present forms that are identical except for orientation and for the tested property; arrays in the center present forms that vary randomly in a second property. Performance of adults and children appears on the right (after Izard & Spelke, 2009, and Izard *et al.*, 2011b).

**a. Numerical WOC with actions**



**b. Spatial WOC**



**Figure 10.** Arrays used to elicit numerical and spatial descriptions from IC, a deaf adolescent who communicated only by means of a non-conventional gestural system. (a) When IC viewed successive arrays presenting the same kinds of objects in different numbers, he spontaneously produced gestures for numbers to distinguish later arrays from earlier ones. (b) When IC viewed successive arrays of the same kinds of objects in different spatial arrangements, he never produced gestures for these relationships, even after repeated prompting and modeling of such gestures.