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RIVER BASINS: WATER AND COMPLEX ADAPTIVE SYSTEMS

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1. INTRODUCTION

The study of empirical and theoretical evidence on the structure of river basins suggests somewhat disparate linkages of features shown by natural networks, whether living or inanimate. Part of the similarity might indeed be spurious, as certain topological properties, for instance, tend to be reproduced by a variety of tree-like forms regardless of wide – even visible at eyeball – differences of other nature. If indeed distinctive of different forms, linkages of geometrical or topological features could be important because the embedded function – one speculates, in fact, that it should be possible to precisely the geometrical features as the signatures of the evolutionary properties that molded them. A comparative study of networks thus possibly relates to the general theme of the PAS Workshop, and deeply concerns hydrologic research.

Of great importance, in this context, is the outstanding progress of the observations (and their objective manipulations) recently achieved for the description of river basins from the scale of a few meters (down to O(1) m) to several thousands of kilometers. In this respect, river basins constitute one of the most reliable and fascinating laboratories for the observation of how Nature works across a wide range of scales (Rodríguez-Iturbe and Rinaldo, 1997). Only the solid reference to natural observation in very particular cases, in fact, allows the search for consilient mechanisms aiming at general rules. On this basis alone the importance of the study of the river basin can hardly be overestimated.

Two problems, both related to the dynamic origin of natural forms, have interested scientists for a long time. One is the fundamental dynamic reason behind Mandelbrot's (1977) observation that many structures in nature –

such as river networks or coastlines – are fractal, i.e. looking 'alike' on many length scales. The other is the origin of the widespread phenomenon called *1/f* noise, originally referring to the particular property of a time signal, be it the light curve of a quasar or the record of river flows, which has components of all durations, i.e. without a characteristic time scale. The name *1/f* refers to the power law decay with exponent –1 of the power spectrum S(f) of certain self-affine records and is conventionally extended to all signals whose spectrum decays algebraically, i.e. $S(f) \sim f^{-\alpha}$. Power-law decay of spectral features is also viewed as a fingerprint of spatially scale-free behaviour, commonly defined as critical. In this framework criticality of a system postulates the capability of communicating information throughout its entire structure, connections being distributed on all scales. The causes and the possible relation for the abundance found in nature of fractal forms and *1/f* signals have puzzled scientists for years. Per Bak (e.g. 1996) and collaborators have addressed the link of the above problems, suggesting that the abundance in nature of spatial and temporal scale-free behaviours may reflect a universal tendency of large, driven dynamical systems with many degrees of freedom to evolve into a stable critical state, far from equilibrium, characterized by the absence of characteristic spatial or temporal time scales. The key idea and its successive applications address such universal tendency and bear important implications on our understanding of complex natural processes. The common dynamic denominator underlying fractal growth is now central to our interests in landform evolution.

The resistance to Bak's idea of universality was (and still is in some circles) noteworthy. Science, and geomorphology in particular, is largely committed to the reductionist approach. The reductionist tenet is that if one is capable to dissect and understand the processes to their smallest pieces then the capability to explain the general picture, including complexity, is granted. However, the reductionist approach, affected as it is by the need of specifying so many detailed processes operating in nature and the tuning of many parameters, though suited to describe individual forms, is an unlikely candidate to explain the ubiquity of scaling forms and the recursive characters of processes operating in very different conditions. Are scale-free, recursive characters of the evolution of complex systems tied to the detailed specification of the dynamics? Or, on the contrary, do they appear out of some intrinsic property of the evolution itself? I believe, following Bak, that the invisible hand guiding evolution of large interactive systems should be found in some general properties of the dynamics rather than in some unlikely fine-tuning of its elementary ingredients (Rodríguez-Iturbe and Rinaldo, 1997).

One crucial feature of the organization of fractal structures in large dynamical systems is the power-law structure of the probability distributions characterizing their geometrical properties.

This behaviour, characterized by events and forms of all sizes, is consistent with the fact that many complex systems in nature evolve in an intermittent, burst-like way rather than in a smooth, gradual manner. The distribution of earthquake magnitudes obeys Gutemberg-Richter law (1956) which is a power-law of energy release. Fluctuations in economics also follow power-law distributions with long tails describing intermittent large events, as first elucidated by Mandelbrot's (1963) famous example of the variation of cotton prices. Punctuations dominate biological evolution (Gould and Eldridge, 1993) where many species become extinct and new species appear interrupting periods of stasis. Levy distributions (characterized by algebraic decay of tails, i.e. of the probability of large events) describe mathematically the probabilistic structure of such events. They differ fundamentally from Gaussian distributions – which have exponential decay of tails and therefore vanishing probabilities of large fluctuations – although both are limiting distributions when many independent random variables are added together. In essence, if the distribution of individual events decays sufficiently rapidly, say with non-diverging second moment, the limiting distribution is Gaussian. Thus the largest fluctuations appear because many individual events happen to concert their action in the same direction. If, instead, the individual events have a diverging second moment – or even diverging average size – the limiting distribution could be Levy because its large fluctuations are formed by individual events rather than by the sum of many events. Thus the keywords now are fractal, chaos and power laws (e.g. Schroeder, 1991).

When studying large, catastrophic events in a large system with interacting agents one can try to identify an individual event as the particular source. Rather than the recognition of the achievement of a critical state, a 'Gaussian' observer may discard the event as atypical – as noted by Mandelbrot (1983) when studying the statistics of fluctuations because the remaining events trivially follow Gaussian statistics. A rather common reaction to catastrophic concerted actions is to find specific reasons for large events. Economists tend to look for specific mechanisms for large stock fluctuations, geophysicists look for specific configurations of fault zones leading to catastrophic earthquakes, biologists look for external sources, such as meteors hitting the earth, in order to explain large extinction events, physicists view the large scale structure of the universe as the consequence of some particular dynamics. In essence, as Bak put it, one reluctantly views large events as statistical phenomena.

Bak noted that there is another explanation, unrelated to specific events and embedded in the mechanisms of self-organization into critical states. In such states each large event has a specific source, a particular addition of a grain of sand landing on a specific spot of a sand-pile triggering a large avalanche, the burning of a given tree igniting a large forest fire, the rupture of a fault segment yielding the big earthquake, or the slowing of a particular car starting a giant traffic jam. Nevertheless even if each of the above particular initiating events were prevented, large events would eventually start for some other reason at some other place of the evolving system. In critical systems no local attempt to control large fluctuations can be successful unless for directing events to some other part of the system.

What are the signatures and the origins of the process of self-organization? Bak suggested that self-organized critical systems have one key feature in common: the dynamics is governed by sites with extremal values of the 'signal', be it the slope of a sandpile or the age of the oldest tree in a burning forest, rather than by some average property of the field. In these systems nothing happens before some threshold is reached. When the least stable part of the system reaches its threshold, a burst of activity is triggered in the system yielding minor or major consequences depending on its state. Complexity arises through the unpredictable consequences of the bursts of activity suggesting that the dynamics of Nature may often be driven by atypical, extremal features. This is suggestive, among other things, of Kauffman's example of biological evolution as driven by exceptional mutations leading to species with a superior ability to proliferate or to Bak's example of the introduction of program trading causing the crash of stock prices in October 1987. In both cases a new fact leads to breakthroughs propagating throughout an entire concerted system because it generates chain reactions of global size. Another feature of self-organizing processes is that, in order to have a chance to appear ubiquitously, they must be robust with respect to initial conditions or to the presence of quenched disorder and should not depend on parameter tuning.

A major challenge thus lies in the explanation of the dynamic origin of fractal forms. Considerable efforts have been devoted to define static or dynamic models able to reproduce the statistical characteristics of fluvial patterns, and general concepts like self-organized criticality have been explored in this context (Rinaldo *et al.*, 1993). It should be observed that real drainage basins are not static but usually evolve on extremely long time scales. Nevertheless, statistical properties seem to be preserved during most of the evolutionary process of a basin – most features characterizing the river basin morphology are irrespective of age. Some geomorphological signatures like valley densities (the relative extent of unchanneled concave areas), however, reflect climate changes without appreciable changes in the basic scaling features of aggregated area and length (Rinaldo *et al.*, 1995).

It is worthwhile recalling that it is not appropriate herein to review the theoretical background of landscape evolution models. Yet river networks may be defined by nodes on a regular lattice representing the elevation field, and links determined by steepest descent on the topography whose evolution determines the structure. Thus to find the simplest model that simulates the dynamical evolution of morphologically realistic landscapes and that preserves certain features during evolution is of interest. The reader is referred, for a recent account of the subject, to Rinaldo *et al.* (2006).

'Water making its environment' is thus a variant of the PAS Workshop's title that describes well the proposed contents of my contribution. All this applies to the river basin, carved by water and weathered by other agents, including biological ones. In the case of living, ecological agents one usually talks about complex adaptive systems rather than of critical self-organization (Levin, 1999). Hence my title reflects the forecast that soon solid theoretical and observational relations will link food webs, ecological communities and river drainages both as hierarchical networks and complex adaptive systems (Power and Dietrich, 2002; Muneepeerakul *et al.*, 2006).

2. Of Form and Function

Numerical simulations of the proper equations have shown that landscape evolution is characterized by two distinct time scales. The soil elevations are lowered in a nonuniform way by erosion, causing variations in the drainage directions during the evolution. In a lattice model, at any given time, one may represent the drainage directions at all sites by means of a two dimensional map. After a first characteristic time, some sort of 'freezing' time, the spanning graph determining the drainage directions in the basin does no longer change. Erosion keeps acting on the landscape and changes the soil height, but preserves the drainage structure. The second characteristic time, which is much longer, is the relaxation time at which the profile reaches its stable shape. Because many of the measured quantities, such as the distributions of drained areas and mainstream lengths, depend only on the two dimensional map, the existence of a freezing time much smaller than the relaxation time may provide an explanation for the fact that several statistical properties are found to be almost the same for many rivers, irrespective of their age. Thus the spatial analysis of riverine forms at any time on the geological process makes sense almost in general.

Natural and artificial network patterns, however, show a great variety of forms and functions and many do not show the tree characters (i.e. a unique path from any site to the outlet) that rivers exhibit. One thus wonders what is the basic dynamic reason for radically different forms and functions. Figures 1 to 5 illustrate a sample of the above variety. A reference framework for different types of hydrologic networks is meant to show that departures from trees and tree-like networks spanning a given area commonly arise. A choice of real and abstract structures relevant to hydrology is proposed (Figure 1, see over). Optimal channel networks (OCNs) are described elsewhere in detail (see e.g. Rodríguez-Iturbe and Rinaldo, 1997) and briefly in the next section of this paper. Suffice here to mention that they hold fractal characteristics that are obtained through a specific selection process from which one obtains a rich structure of scaling optimal forms that are known to closely conform to the scaling of real networks even in the case of unrealistic geometric boundaries. To design a very inefficient tree, a non-directed structure is shown (Figure 1d) constrained to be tree-like. This basically corresponds to an algorithm that accepts any change attributed sequentially at random sites of an evolving spanning network – an existing link is disconnected at a random site and rewired randomly to another nearest neighbor provided the change maintains a tree-like structure. Hot OCNs, so called because they correspond to high temperatures of a Metropolis scheme where every spanning tree seeded in the outlet is equally likely, are thus abstract forms meant to reproduce a rather undirected tree. Peano's or Scheidegger's constructs are briefly described in the caption. They are particularly important because many of their geometrical or topological characteristics can be solved exactly.

Loops are also observed in earth landscapes. Powerful or weak tidal forcings (Figures 2 and 3, pages 164-165) introduce preferential scales related to crossovers of processes operating with comparable ranges into an otherwise aggregated pattern. However, deltaic networks arising from the interplay of distributive drainage patterns and of marine and littoral processes can substantially alter the similarity of the parts and the whole. One notes, in fact, that the structure of deltaic networks shows major loops, differently from rivers in runoff-producing areas. Differences in the sinuosity of the branches prevent any detailed statistical similarity of the parts and the whole across scales.

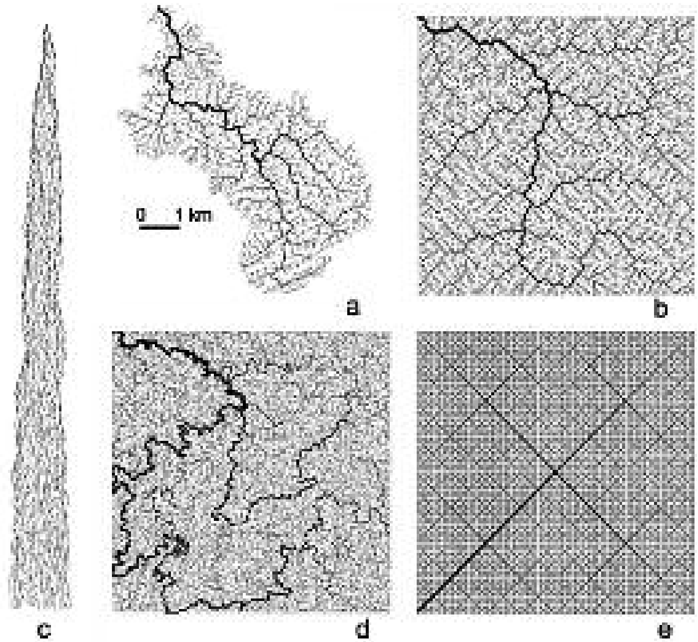

Figure 1. Samples of trees where a unique pattern links any inner site to the outlet of the network: a) A real river network, the Dry Tug Fork (CA), suitably extracted from digital terrain maps. Notice its clear tree-like structure, usual in the runoff production zone of the river basin. Its morphological features (like aggregation and elongation) are typical of fluvial patterns and recurrent 'modules' appear regardless of the scale of total contributing area, such that the parts and the whole are quite similar notwithstanding local signatures of geologic controls, here marked by a fault line clearly visible across the landscape; b) a single-outlet optimal channel network (OCN) selected starting from an arbitrary initial condition by an algorithm accepting random changes only of lowering the total energy dissipation of the system as a whole – thus incapable of reaching the ground state and settling in a local minimum dynamically accessible; c) Scheidegger's construction is generated by a stochastic rule – with even probability, a walker chooses between right or left forward sites only. The model was devised with reference to drainage patterns of an intramontane trench and maps exactly into a model of random aggregation with injection or voter models and also describes the time activity of a self-organized critical (Abelian) avalanche; d) a 'hot' OCN where any arbitrary change randomly assigned to an evolving network is accepted provided it maintains a tree-like form. It is clearly unrealistic fora variety of reasons; e) Peano's construct, where a perfectly recursive rule is adopted to produce a structure whose topological features resemble admirably those of real rivers. It is a deterministic fractal, whose main topological and scaling features, some involving exact multifractals, have been solved analytically. The basic prefractal is a cross seeded in a corner of the square domain that covers the cross and its ensuing iterations. All subsequent subdivisions cut in half each branch to reproduce the prefractal on four, equal subbasins. Here the process is shown at the 11th stage of iteration (from Rinaldo *et al.*, 2006).

Legitimate questions naturally arise by looking at the structures in Figures 1-5: why should loopless trees develop? Are the observed landforms random structures? Are different network configurations equally probable? If not, to what selective pressure do they respond? Are there universal features shown by fluvial landforms, and what is their proper characterization? The fluvial network context alone thus proposes the need for some settlement with respect to the general dynamic origin of network scale invariance, possibly towards an understanding of a general framework for the processes of network growth and selection.

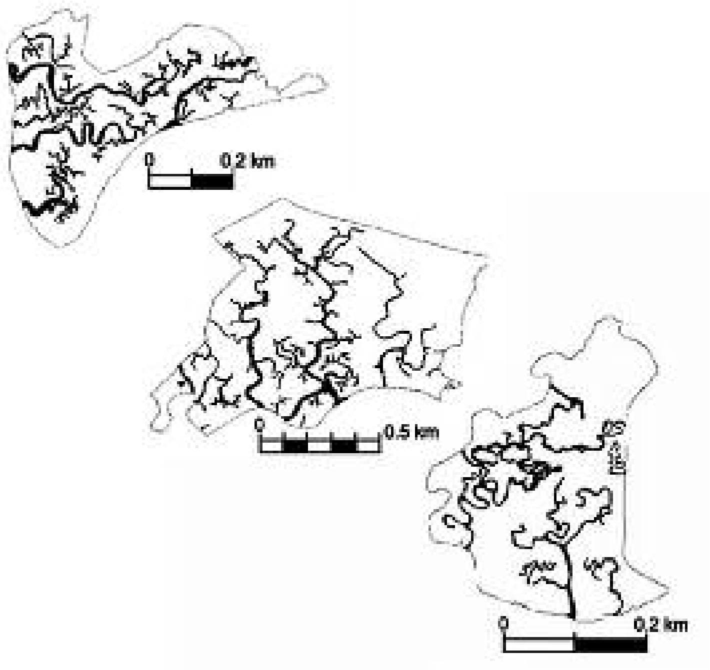

Figure 2. Drainage patterns that form in the tidal landscape are much diverse according to local conditions and develop over a different range of scales. In addition, they may or may not develop loops. Here we show three tidal networks developed within saltmarshes of the lagoon of Venice suitably extracted from remote sensing (Feola *et al.*, 2005). The tidal networks are observed a few km away in space and roughly within the same microtidal range, have similar scales and very different aggregation. Sinuosities of the tidal meanders vary greatly from site to site, possibly reflecting the age of the salt marsh, and the drainage density describing the average distance one has to walk before encountering a creek within a saltmarsh is widely different from site to site (Marani *et al.*, 2003). The presence of loops much depends on local conditions, and is not necessarily affected by flood- or ebb-dominance (after Rinaldo *et al.*, 2006).

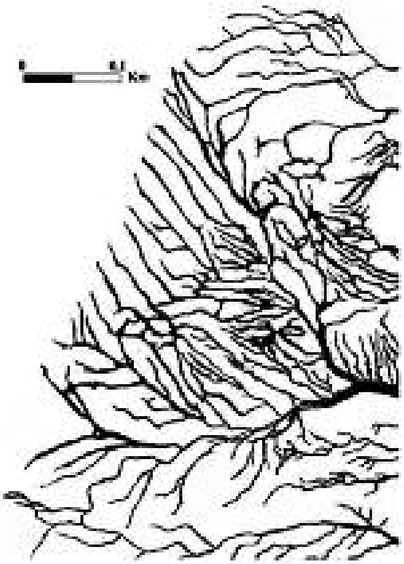

Figure 3. A small-scale drainage pattern developed within a macro-tidal environment, here the Eden estuary in Scotland. Channeled pathways are extracted by suitable digital image processing techniques (courtesy of Enrica Belluco). Here we see an organized sequence of regular ditches reminding of the organization of trenches draining into a complex, looping network structure. Even at eyesight, one catches the lack of a fundamental similarity of the parts and the whole that characterizes river networks (after Rinaldo *et al.*, 2006).

Branching river networks are striking examples of natural fractal patterns which self-organize, despite great diversities in forcing geologic, lithologic, vegetational, climatic and hydrologic factors, into forms showing deep similarities of the parts and the whole across up to six orders of magnitude, and recurrent patterns everywhere (Rodríguez-Iturbe and Rinaldo, 1997). Form and function coevolve. Interestingly, the drainage network in a river basin shows tree-like structures that provide efficient means of transportation for runoff and sediment and show clear evidence of fractal behavior. Numerous efforts to model the production zone of a river (where the system is open, i.e. water is more or less uniformly injected in space and later collected through the structure of the network implying that landscape-forming processes are well defined) have focused on reproducing the statistical characteristics of the drainage network.

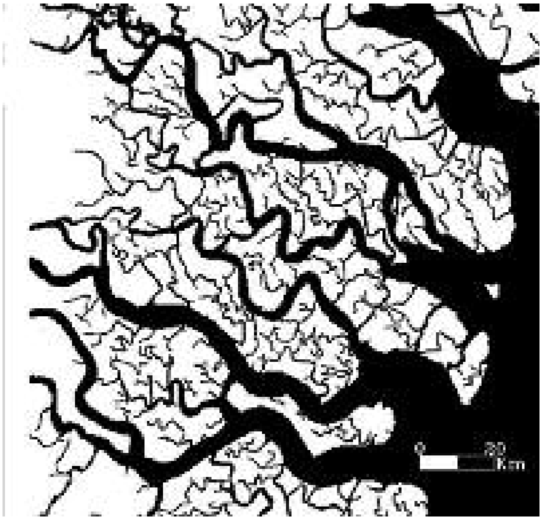

Figure 4. A large-scale, space-born image of the Brahmaputra-Ganges deltaic network. The original NASA image is taken from *http://www.visibleearth.nasa.gov/* and the channelized pattern is extracted via suitable image processing techniques that recognize the spectral signatures of water (courtesy of Enrica Belluco). The complex interplay of the distributional characters typical of deltaic patterns and of the drainage patterns affected by strong tidal forcings (emphasized by the pronounced gradients of channel widths) produces loops appearing on all scales (after Rinaldo *et al.*, 2006).

Much attention has also been paid to the temporal behavior and to the evolution of the topography of the basins, the so-called landscape evolution problem.

Our observational capabilities are also noteworthy. Accurate data describing the fluvial landscape across scales (covering up to 5 orders of magnitude) are extracted from digital terrain maps remotely collected and objectively manipulated. Raw data consist of discretized elevation fields z_i on a lattice. The drainage network is determined assigning to each site i a drainage direction through steepest descent at i, i.e. along ∇z_i. Multiple flow directions in topographically convex sites, and their derived hydrologic quantities, are also easily tackled. Many geomorphological features are then derived and analyzed. To each pixel i (the unit area on the lattice) one can associate a variable that gives the number of pixels draining through i i.e. following the flow directions. This quantity represents the total drainage area (total contributing, or accumulated, area) A_i at the point i, expressed e.g. in pixel units, via

$$A_i = \Sigma_j \, w_{j,i} \, A_j + 1$$

where $w_{j,i}$ is the element of an adjacency matrix, i.e. it is 1 if $j \rightarrow i$ and 0 otherwise. Here 1 represents the unit area of the 'pixel' unit that discretizes the surface. In the case of uniform rainfall injection, a_i provides a measure of the flow at point i.

Drainage directions determine uniquely network lengths. Downstream lengths (i.e. from a site to the outlet following the largest topographic gradient, i.e. steepest descent) can be computed easily to derive their distributions which clearly show the characters of finite-size scaling (Maritan *et al.*, 1996). The upstream length is defined as the distance, measured along the stream, from the farthest source draining into i. Overall, channelized patterns are now reliably extracted from topographic fields through the exceedence of geomorphological thresholds, and have thus much improved our ability to describe objectively natural forms over several orders of magnitude. Large-scale observations have allowed thorough comparisons across scales defining fractal river basins. One outstanding example of fractal relation is Hack's law relating the upstream length l_i at a given position i to the total cumulative area a_i at that position, seen quite early as a signature of the fractal geometry of nature. Contributing area A_i at any point is related to the gradient of the height (the topographic slope) of the landscape at that point: $\nabla z_i \sim A_i^{g-1}$ with a numerical value of g around 0.5 (e.g. Montgomery and Dietrich, 1992). This slope-discharge relation proves a powerful synthesis of the local physics. The distributions of cumulative areas a and upstream lengths l are characterized by power law distributions (with the expected finite size corrections) with exponents in the narrow and related (Rinaldo *et al.*, 1999) ranges 1.40-1.46 and 1.67-1.85, respectively. It is particularly revealing, in this context, that the finite-size scaling ansatz provides a most stringent observational proof of self-similarity.

Figure 5 (see page 233) shows a typical case of finite-size scaling analysis for total contributing areas within a natural river basin. Details are in the caption.

Scaling in the river basin has been documented in many other geomorphological indicators and exact limit scalings identified (Banavar *et al.*, 1999), making the case for the fractal geometry of Nature particularly compelling (Rinaldo *et al.*, 1999). Further proofs have been found by the striking invariance of probability distributions of length and area under coarse graining of the elevation field (Rodríguez-Iturbe and Rinaldo, 1997). The case of rivers is thus a solid starting point for other queries about the possible consilience of natural mechanisms that involve the signatures of complex adaptive systems.

4. Optimal Channel Networks, Loopless Structures & the Dynamics of Fractal Growth

It has been suggested (Rodríguez-Iturbe *et al.*, 1992) that optimal networks are spanning loopless configurations only under precise physical requirements that arise under the constraints imposed by continuity. In the case of rivers, every spanning tree proves a local minimum of total energy dissipation. This is stated in a theorem form applicable to generic networks, suggesting that other branching structures occurring in Nature (e.g. scale-free and looping) may possibly arise through optimality to different selective pressures.

In this section we review the basis for the claim that tree-like fluvial structures are a natural by-product of some optimization of form and function peculiar to the physics of rivers (Rodríguez-Iturbe *et al.*, 1992 a-c) and its implications (Rinaldo *et al.*, 1992, 1996, 1999). The OCN model was originally based on the ansatz that configurations occurring in nature are those that minimize a functional describing the dissipated energy and on the derivation of an explicit form for such a functional. A major step was the later proof (Banavar *et al.*, 1997, 2001) that optimal networks are exactly related to the stationary solutions of the basic landscape evolution equation. In particular, any configuration that minimizes total energy dissipation, within the framework of general dynamical rules, corresponds, through the slope discharge relation, to an elevation field that is a stationary solution of the basic landscape evolution equation. Thus spanning, loopless network configurations characterized by minimum energy dissipation E are obtained by selecting the configuration, say s, that minimizes:

$$E(s)=\Sigma_i A_i^{\gamma}$$

where i spans the lattice and a_i and γ are as defined above. It is crucial, as we shall see later, that one has $\gamma<1$ directly from the physics of the problem.

The global minimum (i.e. the ground state) of the functional for E(s) is exactly characterized by known mean field exponents (Maritan *et al.*, 1996b), and one might expect to approach this mean field behavior on trying to reach stable local minima on annealing of the system. This is in fact the case. The proof of the above is not trivial. We thus maintain, following proper analyses based on observational data, that the drainage basin can be reconstructed using the rule of steepest descent i.e. the flux in a point has the direction of the maximum gradient of the elevation field (the direction towards the lowest among all its nearest neighbors). Moreover, the channelized part of the landscape is necessarily (but not sufficiently) identified by concave areas where the above assumption holds strictly. One can thus uniquely associate any landscape with an oriented spanning graph on the lattice, i.e. an oriented loopless graph passing through each point. Identifying the flux in a point with the total area drained in that point, one can reconstruct the field of fluxes corresponding to a given oriented spanning graph. From the fluxes, a new field of elevation can be defined (Rinaldo *et al.*, 2006). It should be noted that we must limit our attention to patterns embedded in runoff-producing areas within the above framework to confine the problem within tractable limits. As we shall see, theoretical matters are already complex enough even in this oversimplified framework, and the patterns produced surprisingly similar to those observed in nature.

Note also that we wish to emphasize the dependence on the configuration s, an oriented spanning graph associated with the landscape topography z through its gradients ∇z. An interesting question is how networks resulting from the erosional dynamics are related to the optimal networks arising from the minimization of the dissipated energy. Specifically, we require that any landscape reconstructed from an optimal configuration using the slope-discharge relation is a stationary solution of the evolution equation. Superficially, this may seem to be a trivial fact because the relation between gradients and flows is verified by construction, but one should notice that the slope discharge relation alone does not implies stationarity, because the flow may not be (and in general is not) in the direction of the steepest descent in the reconstructed landscape. Thus optimal channel networks consist of the configurations s which are local minima of E(s) in the sense specified below: two configurations s and s′ are close if one can move from one to the other just by changing the direction of a single link (i.e. the set of links $s \subset s'$ represent a graph with a single loop). A configuration s is said a local minimum of the functional E(s) if each of the close configurations s′ corresponds to greater energy expended. Note that not all changes are allowed in the sense that the new graph again needs to be loopless. Thus a local minimum is a stable configuration under a single link flip dynamics, i.e. a dynamics in which only one link can be flipped at a given time, and is flipped only when the move does not create loops and decreases the functional E. Any elevation field thus obtained by enforcing the slope-area relation to a configuration minimizing at least locally E(s) is a stationary solution, i.e. the landscape reconstructed from an optimal drainage network with the slope-discharge rule is consistent with the fact that the flow must follow steepest descent.

The OCN model has been thoroughly analyzed (Maritan *et al.*, 1996b; Banavar *et al.*, 2001). In particular, the scaling behavior of the global minimum has been worked out analytically and it has been found to yield mean field exponents. Interestingly, local minima also exhibit critical behavior but are characterized by different nontrivial scaling exponents of key probability distributions describing e.g. drained area, channelled length, elongation (e.g. Rodríguez-Iturbe and Rinaldo, 1997).

Figures 6 a-c, that complement Figure 1c, show examples of local and global minima of OCNs (here chosen in a multiple-outlet configuration). Figure 6a shows the result obtained by 'Eden' growth generated by a self-avoiding random walk, which is known to lead to suboptimal structures (Rodríguez-Iturbe and Rinaldo, 1997). It is interesting to use Eden structures as benchmarks because their chance-dominated selection principle (no necessity is implied by the random-walk dynamics, and tree-like structures are selected because of the self-avoiding nature imposed on the process) because such structures were initially thought of as capturing the essentials of natural selection. That turned out to be an artifact of non-distinctive tests of the network structure, nicely termed the 'statistical inevitability' of Horton's laws. Indeed if topological measures alone (e.g. Horton numbers, Tokunaga matrices) are used to sort out the fine properties of networks, one can be hugely misled into finding spurious similarities with natural forms, as one would sometimes safely conclude even at eyesight: compare e.g. Figure 6a with b – topological features like e.g. those based on Strahler's ordering like Horton ratios or Tokunaga's matrices are indistinguishable in the two cases shown (Rinaldo *et al.*, 1999). Yet these are very different networks, as (linked) scaling exponents of areas, lengths and elongation clearly reveal. If eyesight and common sense would not suffice, exact proofs are available, like in the case of Peano's basin (Figure 1e) where topological measures match perfectly those of real basins and of OCNs, but fail to satisfy the strict requirements of aggregation and elongation. More subtle but equally clear is the failure of random walk type models or topologically random networks to comply with exhaustive comparisons (Rinaldo *et al.*, 1999). Notice that the latter models were extremely influential in suggesting that chance alone was behind the recurrence of natural patterns, because of the equal likelihood of any network configurations implied by the topologically random model. Instead their purported similarity with natural patterns is now seen as an artifact of lenient comparative tools, and the statistical properties and 'laws' derived in that context are almost inevitable for spanning trees.

Necessity is instead at work in the selection of natural networks. Figure 6b shows a local minimum of E(s), whose fine features match perfectly those found in Nature (Rinaldo *et al.*, 1999) including a power law in the distribution of cumulative areas with exponent $g=0.43\pm0.01$ (compare e.g. with Figure 5). These results, (b), are obtained moving from an initial configuration s. A site is then chosen at random, and the configuration is perturbed by disconnecting a link, which is reoriented to produce a new configuration s′. If the new configuration lowers total energy dissipation i.e. $E(s') < E(s)$, the change is accepted and the procedure is restarted. Figure 5 (c) is obtained through the same procedure used to obtain (b), where an annealing procedure has been implemented, i.e. unfavorable changes may also be accepted with probability proportional to $e^{-(E(s)-E(s'))/T}$ where T assumes the role of temperature in a gas or a spin glass. It is rather instructive to compare Figure 5(b) with (c), where a ground state is reached by very careful annealing using a schedule of slowly decreasing temperatures. This state is characterized by mean field scaling exponents (here matched perfectly), and overall all too regular and straight to reproduce, even at eyesight, the irregular and yet repetitive vagaries of Nature.

Several random constructs have been thoroughly analyzed, in a few cases through exact results, comparing them with optimal ones obtained through minimization of total energy dissipation (e.g. Rinaldo *et al.*, 1999). Random network forms range from self-avoiding random walks like Eden growth patterns, topologically random or Leopold-Langbein constructions, to the so-called Scheidegger network (Figure 1) which is a directed random aggregation pattern with injection. Deterministic fractals like Peano's networks have been also exactly analyzed. Thus many misleading similarities are inferred from the matching of topological measures like Horton's ratios. These turn out to be too lenient measures, as they occur almost inevitably for spanning loopless networks and thereby do not distinguish the structure of the aggregations patterns. On this basis alone it was shown that topolog-

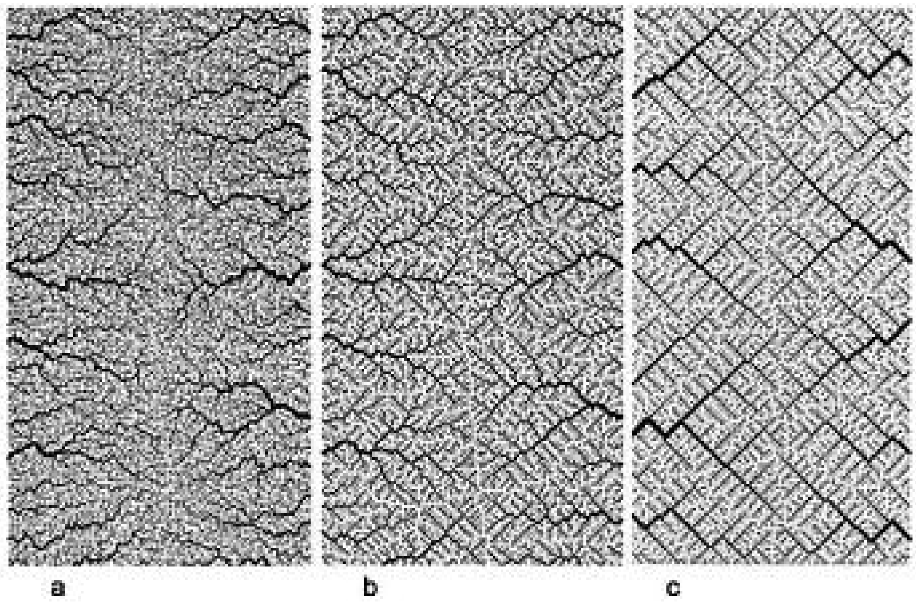

Figure 6. Multiple-outlet networks obtained in the same rectangular domain by: (a) Eden growth patterns of self-avoiding random walks filling the domain; (b) an imperfect optimal channel network (OCN) leading to a local minimum of total energy dissipation. Note that OCNs bear long-lived signatures of the initial condition owing to the myopic search procedure, but actually reproduce perfectly the aggregation and elongation structure seen in real river landscapes; and (c) ground-state OCNs obtained through simulated annealing using a very slow schedule of decreasing temperatures. The reaching of the ground state is confirmed by the matching of the exact mean field exponents with those calculated for (c) (Rinaldo *et al.*, 2006).

ical similarities are to be interpreted as necessary, rather than sufficient, conditions for comparison of network structures. A distinctive comparison of network structures stems from the matching of several scaling exponents which characterize the finite-size scaling forms of the distributions of length, aggregated area and elongation. At times perfect matching of aggregation (underlined by Horton's ratios of bifurcation, length and area indistinguishable from those observed in natural structures) proves inconsistent with the structure of channelled lengths, like in the case of self-avoiding random walks. Moreover, suboptimal networks, that is, those derived by imperfect search of the type perceived as dynamically feasible, match all the features of the networks observed in the fluvial landscape, and thus pass the most thorough screening differently from all chance-dominated constructs. The hydrologic context thus suggests a case for optimal selection of network structures in Nature.

I shall now finally define precisely the selective advantags of trees in the fluvial physics (Banavar *et al.*, 2001). Consider a square lattice. Fix an orientation for all lattice bonds. On each bond b a flux J_b is defined. Assume that $J_b>0$ if it is flowing along an assigned orientation. Uniform (unit) injection is equivalent to the set of constraints $\nabla \cdot J=1$, i.e. a discrete version of the divergence, and is a measure of the net outflow from a site:

$$\nabla \cdot J=\Sigma_{b \in x} J_b \ \theta(b,x)=1$$

where: the unit value is the model injection, constant for every node in the simplest case; b spans all bonds (links) concurring on node x, and q(b,x)=1 (-1) if b is oriented outward (inward) node x. Any local minimum of the function:

$$E=\Sigma_b J_b^{\gamma}$$

when $0<\gamma<1$, corresponds to $J_b \neq 0$ only on the bonds of a spanning tree. Note that the assimilation $J \sim A$ is commonplace in hydrology, for a variety of empirical and theoretical reasons (Rodríguez-Iturbe and Rinaldo, 1997) – thus the above equation is exactly that defining an OCN. The main point is in the proof that the networks that correspond to local minima of the dissipated energy are loopless and tree-like. The tree must be spanning due to the above constraints: one cannot have $J_b=0$ for all b's connected to a site so that there must be at least one outlet from each site x. Some site (or sites) must also be declared to be the global outlet.

Figure 7 illustrates an extremely simple example with just four sites: (a) shows the setup for the elementary 4-bond network. The dot is the outlet. Here the current a is taken as the parameter regulating the entire distribution of fluxes owing to continuity; (b) illustrates the only loopless configurations of the system generated by integer values of a; (c) shows the plot of the function E vs a from the following Equation with g=0.5:

$$E=a^{\gamma} +(a+1)^{\gamma}+(1-a)^{\gamma}+(2-a)^{\gamma}$$

which is derived from the above equations. In particular, (b) shows the plot E(a) where one notices that there are local minima in correspondence with one of the four currents being zero (a=2, 1, 0, -1), corresponding to the four trees shown in Figure 3c. The proof for the general case is elsewhere (Banavar *et al.*, 2001).

Figure 7 (c) shows the function E versus a plotted for various values of γ (specifically, for γ=0.25, 0.5, 1 and 2). Note that for γ=1 all directed (with the currents going in the positive directions) configurations, loopless or not, have the same energy. The case γ=2 corresponds to the resistor network case for which there is just one minimum at a=1/2. Note that since there is one unknown current for each bond and one continuity equation for each site the number of independent variables is given by the number of bonds minus the number of sites (excluding the outlet), which for the simple topologies considered is equal to the number of elementary loops (this is a particular case of the Euler theorem).

The general proof is beyond the scope of this paper and given elsewhere (Banavar *et al.*, 2001) for an arbitrary graph, where the number of independent loops is given by the number of bonds minus the number of sites plus the number of connected components. Note that for the particular case where the graph must be a spanning structure the number of connected components is unit.

Obviously, for the set of dynamical rules postulated above, the energy landscape is riddled with a large number of local minima characterized by

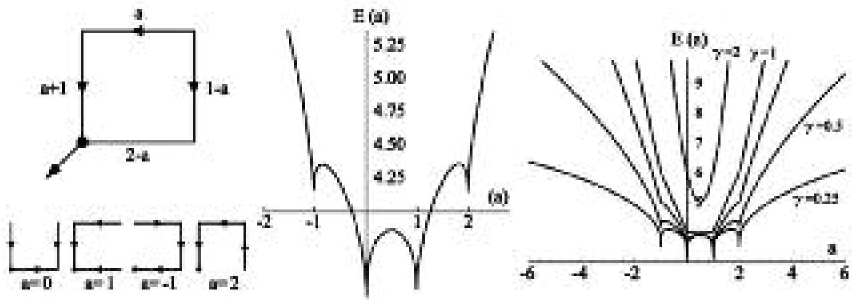

Figure 7. The 4-bond lattice. (Top left) the 4-node arrangement, with indications on the currents that respect continuity (note that a unit flux is injected at each node); (bottom left) the only for possible trees correspond to the cases a=0, -1, 1, 2. (Center) The energy function E(a) vs. a; (Right) Energy functions E(a) vs. a for the cases g=0.5, 0.75, 1, 2 (after Banavar *et al.*, 2001).

a range of similar values of E. In single realizations, boundary and initial conditions affect the feasible (i.e. dynamically accessible) optimal state to different degrees depending on their constraining power. This fact matches the observation that scaling exponents are coherently linked in a range of values, narrow enough but significantly different from the ground state (see e.g. Rodríguez-Iturbe and Rinaldo, 1997). The truly important implications are twofold: one one side, in fact, all local optima are trees; on the other, imperfect optimal search procedures are capable of obtaining suboptimal networks which nevertheless prove statistically indistinguishable from the forms observed in nature and quite different from the absolute minima. Indeed we believe that the worse energetic performance and yet the better representation of the patterns of Nature are thought of as mimicking the myopic tinkering of evolutionary processes.

The rules investigated above also suggest that the convexity of the function defining the selective advantage of different hydrologic network structures matters. In the case of fluvial basins, the basic concavity of the energy E is provided directly by the physics of the landscape evolution problem. It has been studied whether similar principles apply to the selection of different network structures by natural processes of different nature. While a detailed account is given elsewhere (Rinaldo *et al.*, 2006), suffice here to mention that a class of optimal models evolved by local rules and chosen according to global properties of the aggregate yields unexpected behavior in the transition from different types of optimal topologies. Random or scale-free arrangements (and a variety of in-betweens like small-world constructs) are then seen as particular cases emerging from selective pressures towards connectivity and/or directedness.

8. Conclusions

The results I have analyzed here show consistently that selective criteria blend chance and necessity as dynamic origins of recurrent network patterns seen in the river basin. The role of selective pressures as a possible cause of emergence of observed features has been reviewed both in the cases of loopless and of looping networks. In the latter case we show why Nature distinctively favors tree-like shapes in the particular dynamic environments of which landscape evolution carved by water is a particular case.

My main conclusion is that the emergence of the structural properties observed in natural network patterns may not be necessarily due to embedded rules for growth, but may rather reflect the interplay of dynamic mechanisms with an evolutionary selective process. This has implications, of course, for hydrologic research because many landforms originated by the collection or the distribution of hydrologic runoff (from riverine to tidal or deltaic patterns) might indeed be classified according to the compliance to the above mechanisms. Whether the above has somewhat more general relevance to Nature's mechanisms, it remains to be seen as more and more we need to understand how the complex biosphere has emerged from natural selection and other forces operating at small scales.

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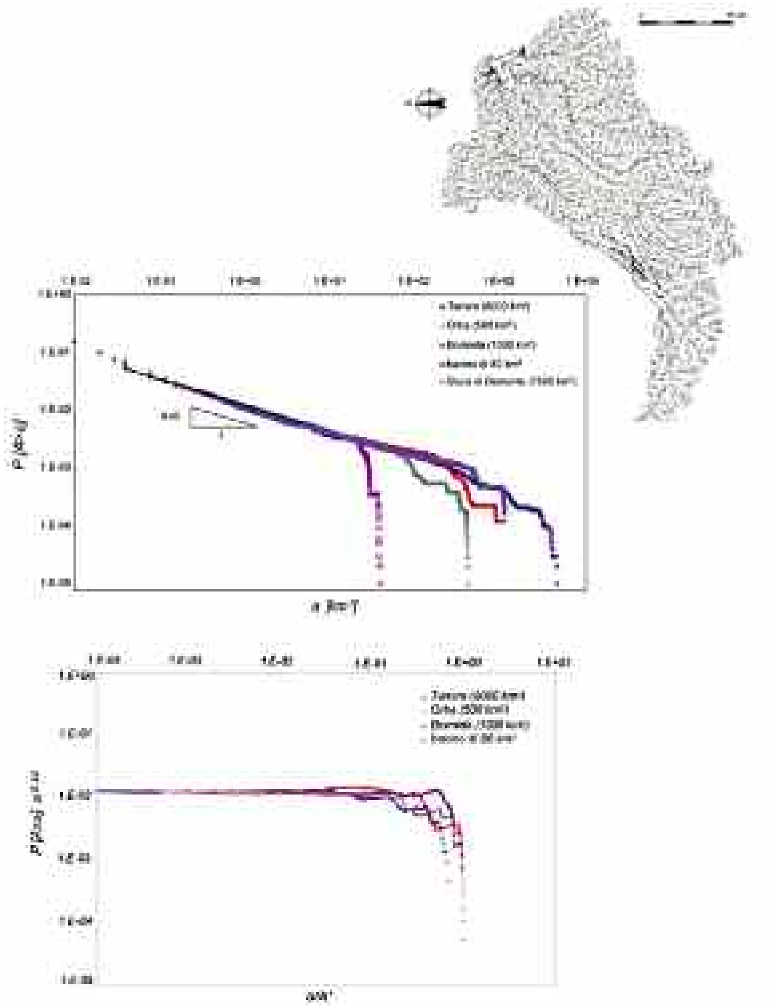

Figure 5. Power-law distribution of total cumulative area (top left) for the 8000 km^2 Tanaro river basin (top right). Here the cumulative distribution $P(A \varnothing a) \sim a^{g-1}$ (which, of course, implies that the pdf scales like a power law with exponent g) is computed for 4 subbasins of different maximum size A* (Tanaro, 8000 km^2; Orba, 500 km^2; Bormida 1300 km^2; and a small unnamed basin, 80 km^2). The similarity of the basins insure that in the range of areas where no cutoff effect is perceived, one obtains a remarkably coherent estimate of g~0.45. The distributions collapse into a single curve, however, once plotted as $P(A \varnothing a)\ a^{0.45}$ vs a/A* (bottom). This suggests a finite-size scaling ansatz for the area distribution, and a strong empirical proof of self-similarity of form in the river basin. Needless to say, the above observation is confirmed by ubiquitous empirical evidence regardless of geology, climate, vegetation and exposed lithology (Rodríguez-Iturbe and Rinaldo, 1997).