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HEISENBERG'S INFLUENCE ON PHYSICS

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I am very happy to have this opportunity to acknowledge my debt to Heisenberg. He provided the clue which set me on the right road and transformed all my work on atomic theory.

The story of Heisenberg's contribution to physics is largely the same as the story of the discovery of quantum mechanics, the mechanics of the atom. Heisenberg found the key providing a way forward when everyone else was baffled and then played a dominant role in the development. He thus had a tremendous influence on the whole course of atomic physics.

Before Heisenberg, atomic theory was based on Bohr orbits. It was a primitive theory with a very restricted range of applicability, essentially a one-electron theory. Heisenberg replaced it by a general and powerful system of equations forming a whole new mechanics, which was to supercede Newton in the description of the small things of the atomic world.

Werner Heisenberg was born in December 1901. His early life was a good preparation for his great work. His father

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August Heisenberg was Professor of Greek at the University of München. He showed in his work creativity of ideas and appreciation of essentials, without being too much concerned about details. These were mental qualities that were inherited by his son Werner. Also Werner was exposed to the ancient Greek philosophy and became himself much interested in philosophical questions. He read the works of Plato and Democritus and was thus set on the lines of thinking about the ultimate constituents of matter.

The young Heisenberg showed great talent and mathematical ability while still a schoolboy. He was admitted to the University of München in 1920. He applied to attend the seminar of the mathematician F. Lindemann, but was turned down on account of having read work by H. Weyl on relativity, which Lindemann thought was unsuitable for a mathematician. He then applied to the theoretical physicist A. Sommerfeld and was accepted. This was a turning point in his career, setting him on the road of theoretical physics.

The world which Heisenberg entered all devolved around Bohr's theory of the atom. This assumes that an atom can exist in various stationary states with definite energies, and can jump from one to another with emission or absorption of radiation whose frequency is connected with the change of energy. In the stationary states the electrons move in periodic orbits which are subject to the classical laws of Newton, but also have to satisfy some extra conditions called quantum conditions, for which certain constants of the motion are integral multiples of Planck's constant h. These multiples are called quantum numbers.

This theory of Bohr had been greatly developed and made more precise by Sommerfeld. Thus in coming to work with Sommerfeld, Heisenberg was in the best possible place for the further development of atomic theory.

In Heisenberg's first semester Sommerfeld gave him a problem on the Zeeman effect, the effect of a magnetic field

on atomic spectra. Sommerfeld handed to Heisenberg the experimental data and asked Heisenberg to find the explanation in terms of the concepts of the Bohr theory. Heisenberg soon found the answer but, surprisingly, this answer involved half quantum numbers.

This early research showed up Heisenberg's qualities — persistance and boldness. He had to follow up, with precise attention to detail, the consequences of the data provided by experiment, and when they led to a violation of accepted ideas he was not dismayed but boldly proposed a revision of those ideas. Probably only a very young man would have dared it.

When Heisenberg told Sommerfeld of his results, Sommerfeld was shocked. It had been a cardinal feature of the Bohr theory and of Sommerfeld's developments that the quantum numbers should always be integers, and now here was Heisenberg proposing to scrap it. There was a long discussion between them, and eventually Sommerfeld had to concede that Heisenberg was probably right.

We now know that the half quantum numbers arise from the spin of the electron. At that time it was taken for granted that the electron could have no such structure as a spin. It was only five years later (in 1925) that Goudsmit and Uhlenbeck published the idea of electron spin, and then only against the opposition of Lorentz, the originator of the relativistic electron. Lorentz had done calculations, based on energy considerations, which showed electron spin to be impossible.

After two years in München, Heisenberg moved to Göttingen and worked in the institute of Max Born. Here his most important work was done. To begin with, it was concerned with getting a better understanding of the structure of spectra of atoms with several electrons.

The lack of knowledge of the spin of the electron caused a great deal of trouble at that time — in particular, not knowing there were two different states of spin. From experiments one found that for any system of electrons, when one added a further electron to it, the number of stationary states for the combined system was not the number one would expect from quantizing the states of the extra electron according to the Bohr-Sommerfeld rules, but was double that number. The doubling was supposed to arise in some mysterious way from the interaction of the new electron with the previous ones.

Heisenberg made an extensive study of the role of this doubling in atomic spectra. He introduced a special name for it, Zweideutigkeit. It was considered as a *mechanically inexplicable* consequence of the interaction of an electron with other electrons of the atom. I was a research student in Cambridge at that time and I remember very well the big impact that Zweideutigkeit had when it was introduced. A special English word had to be coined for it, *duplexity*.

Here was a definite new idea, quite beyond those of the Bohr orbit theory. With its help one could go a long way towards systematising the regularities of the spectral lines of atoms. It did not really explain anything, but it swept the troubles together and concentrated the mystery into a single principle. One recognised the impossibility of probing more deeply into it by calling it, in Heisenberg's words, *mechanically inexplicable.*

Zweideutigkeit was a temporary excitement which fluttered the world of physicists. It died out when the true explanation in terms of electron spin was discovered. But it showed up very well Heisenberg's powers for grappling with serious difficulties, separating out the part that can be dealt with from the part that cannot. These powers played a great role in Heisenberg's attack on the main problem, which was to obtain accurate equations for the interaction of electrons in an atom.

The Bohr theory was essentially a one-electron theory. There was a separate Bohr orbit for each electron. With several electrons in an atom, how did the Bohr orbits interact? I was working on this problem myself and was thus well able to appreciate how baffling it was. Various people had proposed

ideas for dealing with it, all rather artificial, and were following them up with complicated calculations, with very little success.

Heisenberg saw the need for an entirely new approach, and he was even prepared to give up the Bohr orbits altogether. But it was necessary to find something to replace them. I shall now give you the main steps that led Heisenberg to his success.

Each Bohr orbit is a periodic (or multiply periodic) motion of an electron in an atom. According to the classical ideas, when the atom interacts with electromagnetic radiation the important mathematical quantities entering into the description are the amplitudes C_n of the various modes of oscillation and their frequencies ν_n. The suffix n here denotes the quantum number of the orbit (if several quantum numbers are needed, n stands for the whole set of them).

The whole mathematical theory of the interaction is based on the quantities C_n, ν_n describing the Bohr orbits. Now a Bohr orbit is not observable, so the theory is based on non-observable quantities.

Heisenberg had a definite positivistic philosophy which set him against a theory based on non-observable quantities. So his first step was to eliminate the C_n, ν_n from the theory.

With the C_n and ν_n gone, what is one to use instead of them? In the case of the ν_n, the answer is fairly obvious. One should use the frequencies of the observed spectral lines ν_{nm}, each associated with a jump between two Bohr orbits n and m. How should one replace the C_n?

A hint is provided by an earlier statistical theory of emission and absorption of radiation given by Einstein. Einstein assumed that the intensities of the emission and absorption processes between two stationary states n and m are determined by certain coefficients A_{nm}, B_{nm}, and he deduced relations between them from considerations of statistical equlibrium. (This work led to the laser).

Heisenberg, in a joint paper with Kramers, had set up a new equation for the dispersion of light by atoms, obtained by modifying the classical equation. The amplitudes C_n of the classical theory were replaced by new amplitudes C_{nm} connected with the square roots of Einstein's coefficients and the frequencies ν_n of the Bohr orbits were replaced by the observed spectral frequencies ν_{nm}. The modification was carried out in a logical way and the new dispersion formula appeared reasonable and satisfactory. It satisfied Heisenberg's requirement of not making use of the unobservable quantities C_n and ν_n.

This formula suggested to Heisenberg the general rule that one should always replace the classical amplitudes C_n by new amplitudes C_{nm} connected with the Einstein coefficients, each associated with two stationary states n and m.

The foundations were now set for the big advance. Heisenberg conceived the idea of setting up a dynamical theory entirely in terms of amplitudes like C_{nm} and the frequencies ν_{nm}. The Bohr orbits could then be discarded.

Each of the new quantities is associated with two stationary states, not just one. To write them down in a natural way one would put them as

$$\begin{matrix} C_{11} & C_{12} & C_{13} & \cdot & \cdot & \cdot \\ C_{21} & C_{22} & C_{23} & \cdot & \cdot & \cdot \\ C_{31} & C_{32} & C_{33} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{matrix}$$

Such an array is called a matrix.

Heisenberg's great unifying idea was to consider that one should treat the whole array as a single entity and that one should have equations connecting such entities instead of the usual equations of dynamics. This was a step forward into quite a new type of thought for physicists.

At that time there were mathematicians who were familiar with matrices, but Heisenberg knew none of this and had to work out the algebra for himself. The addition of matrices

is obvious. For the multiplication Heisenberg had to find a natural way of combining the elements so as to correspond to the way the previous coefficients C_n got multiplied in the classical equations. The result was the same rule for matrix multiplication that the mathematicians were already using.

Heisenberg's method involved setting up equations with matrices analogous to the usual equations of kinematics and dynamics. He had not gone very far before he came up against the fact that, with the multiplication of matrices u and v, the order is important. In general uv is not equal to vu. Matrices satisfy non-commutative multiplication.

With u and v representing physical quantities, this result was completely foreign to all the mathematics previously used in the description of nature. It was most disturbing for Heisenberg. But still he was forced to accept it if he was not to abandon his whole method.

Heisenberg obtained the main ideas of his theory while he was in Heligoland in the North Sea. He had gone there for rest and recuperation while suffering from hay fever. Here he escaped from the regular routine of his work and was able to survey the whole problem in a relaxed and unhurried way. These are the ideal conditions for making a great discovery — not while one is working at a desk and rigidly forcing one's thoughts along a direction that one has got heavily involved in.

Heisenberg first worked out a simple example, the anharmonic oscillator. He had to do some extensive calculations to see whether his equations were consistent and worked frenziedly through most of one night before he could get things right. When finally everything fit he realized with great excitement that he had indeed obtained a new insight into the mathematical structure underlying the atomic world.

Heisenberg wrote up a paper and showed it to his professor, Max Born, who found it interesting, but strange, as the concept of electron orbit was eliminated. However, Born sent it in for publication.

Born was an expert on matrices and he continued to study Heisenberg's work, together with Jordan, another of his students at Göttingen, while Heisenberg was absent on a lecturing tour in Holland and in Cambridge. Born and Jordan soon found the simplest case of non-commutation,

$$pq - qp = h/2\pi i, \tag{1}$$

involving a dynamical coordinate q and the conjugate momentum p. Here h is Planck's constant. This equation is fundamental in the new theory.

With Heisenberg's return the three of them joined forces and worked intensively on developing the new mechanics. And thus non-commutative algebra was launched into the world of physics.

It is really astonishing that such an extraordinary idea as non-commutative algebra should prove a useful tool for the description of nature. It is so remote from common sense, that one wonders how any human mind, even that of a genius, could ever have thought of it. Certainly not by any direct attack. It could only come by a circuitous route where one was guided all along the way by sound basic principles, strongly backed up by considerable knowledge of experimental results.

You see how Heisenberg was started on this route by his philosophy requiring him to discard unobservable quantities from theory. He had to find suitable observable quantities to replace them. Then he had to find how the observable quantities fit together naturally into equations. This led him to mathematical concepts new to physics, namely matrices. The handling of these matrices then forced him to the new algebra.

The only discovery in recent times comparable with Heisenberg's discovery of the need for non-commutative algebra is Einstein's discovery of the need for curved space. Both discoveries involved a direct denial of what was previously accepted as obvious and both required the introduction of a new kind of mathematics into physics. But the change introduced by

Einstein was not such a strange one conceptually. The possibility of curved space was not so hard to think of and had already been considered by Riemann, who laid the mathematical foundations for it in the last century. Heisenberg's non-commutative algebra was much more astonishing.

Once Heisenberg had sparked off the idea of non-commutative algebra, the whole course of development of atomic theory was altered. It was just the clue that had been missing for so long. Many people now joined in the work and progress became very rapid. Everyone was working with the new theory and enjoying the vast opportunities that it opened up for fresh attacks on problems that had long been perplexing and baffling. It was the beginning of a golden age in theoretical physics, which has never been equalled before or since. Truly a remarkable transformation for one man to have brought about.

The Bohr orbits were superceded, although of course the more fundamental Bohr idea of stationary states for an atom was retained. A new mechanics was built up, called *matrix mechanics*. It was found possible to make its equations very similar to those of the old Newtonian mechanics, especially when the latter were expressed in the Hamiltonian form. One just had to generalize them to bring in the non-commutation. It was an exciting game to play.

The development of the new theory was most rapid on the mathematical side. But this did not give a complete theory. It just gave equations connecting abstract non-commuting quantities which in some way « represented » physical variables. To complete the theory one had to find a method of attaching these non-commuting quantities to numbers, which could then be compared with experiments. How to do this was not at all evident. With uv not equal to vu, how can we attach numbers to u and v? Whatever numbers we give them, uv will always equal vu.

In the early examples one just calculated energy levels and was very happy to get results in agreement with observation.

But this was a very limited application and not enough to give a complete theory.

The development of the general physical interpretation was very much assisted by some fundamental work by E. Schrödinger. Schrödinger was working independently of Heisenberg, following up some ideas of L. de Broglie of associating waves with particles in an analogous way to the association of light-waves with photons. He succeeded in building up a theory for calculating atomic spectra. It was called *wave mechanics*. So we then had two successful theories of the atom, matrix mechanics and wave mechanics. This was an excess of riches.

But Schrödinger soon found that the two kinds of mechanics were really equivalent, and one could pass from one to the other by a mathematical transformation. The basis of the equivalence was that the Schrödinger theory employed differential operators, which also obey non-commutative algebra, so that they could be connected up with matrices satisfying the same formal relations.

Schrödinger set up his ideas quite independent of Heisenberg and could have carried on even if Heisenberg had not existed. One may speculate on how atomic theory would have developed in that case. Schrödinger did not have the powerful idea of non-commutative dynamical variables, but he was well started on a road that must have led to it eventually.

Heisenberg's and Schrödinger's ideas were thus united and became just two forms of the same theory, quantum mechanics. It provided a broad basis for seeking a general physical interpretation.

In May 1926 Heisenberg moved to Copenhagen to become Bohr's assistant. Copenhagen became the centre for discussions on the physical understanding of the new theory. Diverse viewpoints were expressed by different people. The main argument was the controversy between Bohr and Heisenberg on the one hand, who retained the idea of quantum jumps, and

Schrödinger on the other, who wanted his waves to provide a continuous description of atomic events.

Schrödinger, with his new approach, had brought also a new interpretation for quantum mechanics. He considered that the radiation that is emitted or absorbed by an atom comes from the interference of the waves associated by his mechanics with the upper and lower states of the atom. The emission or absorption would then be a continuous process and not connected with a jump. But Schrödinger's picture was a specialized one and he had difficulty extending it to other atomic processes.

Bohr and Heisenberg persisted in the need for quantum jumps. In fact they were basic to Heisenberg's matrices. There were intensive discussions during a visit of Schrödinger to Copenhagen, but neither side was able to convince the other.

With Schrödinger's departure, there were still differences in the points of view of Bohr and Heisenberg, leading to much further discussion. In particular Bohr was concerned with how the idea of particles could be reconciled with Schrödinger's waves. He emphasized the duality of the wave and particle concepts. Heisenberg kept closer to the mathematical description.

In the meantime, mathematical developments led to the possibility of calculating probabilities in a general way. One could calculate the probabilities needed to describe the results of collision processes, and more generally the probability of any dynamical variables having specified values, subject only to the condition that these dynamical variables must commute. It became clear that such probabilities comprised *all the precise information that quantum mechanics could provide.*

During a temporary absence of Bohr, when Heisenberg was left undisturbed for a time, he made a new attack on the question of just what one can observe about the motion of an electron. He realized that observations are never exact. There is always an uncertainty in the result. A brief calculation showed him that such a situation can be handled mathematically. If one is observing a coordinate q, the result will be a number lying probably within a certain range Δq. Similarly if one observes the momentum p, the result will be a number lying probably within a range Δp. Heisenberg was able to deduce from the equation (1) connecting q and p, that if one observes both q and p, under the most favourable circumstances that are theoretically possible,

$$\Delta q \; \Delta p = h/4\pi.$$

This is a condition limiting the accuracies of measurement. One cannot escape from it. It is fundamental for quantum theory. There is no limit to the accuracy with which q may be fixed. Δq may be as small as we please. But then Δp is correspondingly large.

Here is a result which is easy to visualize. It is called Heisenberg's Principle of Indeterminacy. People often take it as the cornerstone of quantum mechanics. But it is not really so, because it is not a precise equation, but only a statement about inaccuracies. The real cornerstone is the precise equation (1) and all its mathematical consequences, leading to the calculation of probabilities.

Bohr took up this Principle of Indeterminacy and generalized it to a Principle of Complementarity. There are complementary quantities, such that an observation of one precludes the observation of the other. For example, the waves and particles are complementary. This applies not only to physics, but to more general fields of knowledge, and Bohr built it up to a wide philosophical doctrine.

Bohr and Heisenberg were then in agreement about the physical interpretation of quantum mechanics. The Principle of Indeterminacy made it clear that the probabilities provided by the mathematics comprise all the information that one can get from quantum theory. One has to renounce the determi-

nacy of the old Newtonian mechanics. This became known as the interpretation according to the Copenhagen school.

Since the probabilities are all that the experimenters want to know, most physicists were happy with this situation. However there remained some, led by Schrödinger and Einstein, who were unable to accept it unreservedly, and clung to the hope of a return to the determinacy of the classical theory.

The new quantum mechanics of Heisenberg and Schrödinger showed itself to be a very powerful tool for investigating problems in the atomic world. So much so that people thought for a time it would lead to the solution of all atomic problems. But after a few years serious difficulties began to show up.

Quantum mechanics is a non-relativistic theory. Its equations (except in certain elementary cases) refer to one particular time axis, instead of equally to all time axes as Einstein's special theory of relativity demands. If one applies it to interacting particles and tries to make the theory relativistic, one is led to equations that have no solution. This holds even for the interaction of electrons and photons, the two particles that we know most about, as was studied in detail in a paper by Heisenberg and Pauli.

Heisenberg tackled this problem in his characteristic manner. He proposed that one should concentrate on the observable quantities, a procedure that had previously led him to his great success.

A relativistic theory is important when one is dealing with high-velocity or high-energy particles. The experiments with such particles are of the nature of collision experiments and the observations are the probabilities of various events occuring with given initial conditions.

Heisenberg supposed that these probabilities would each be given by an amplitude like the matrix elements of his original matrix mechanics. These amplitudes would all combine into a single matrix, which was called the S-matrix. A knowledge of the S-matrix would provide the connection between any initial state and any final state of a collision process. A theory that allows one to calculate the S-matrix would give all the scattering probabilities and thus all the information that experimenters need.

Various conditions can be imposed on the S-matrix, but they are not sufficient to determine it. The problem of finding the S-matrix is still unsolved, after more than 40 years of intensive work by the world's physicists.

Heisenberg, with his introduction of the importance of the S-matrix, has provided the framework for relativistic quantum mechanics, but one does not know how to fill it in. One lacks the equations of motion that provide the detailed information that we have with non-relativistic quantum mechanics.

But still the concept of the S-matrix is most useful. The observations of particular collision processes give information about some of its elements. This can be fitted into the framework, and thus our knowledge of the S-matrix is steadily increasing.

Another idea that Heisenberg has suggested is that there may be a fundamental length λ playing an important role in nature, comparable to that of Planck's constant h. When lengths comparable to λ are involved, our ordinary ideas of space break down. This line of thinking led to the expectation that when such small lengths are involved, corresponding to very high energies, there should be physical reactions taking place leading to the simultaneous creation of many particles. But still no general mathematical theory on these lines has yet been built up.

Heisenberg in later life was much concerned about the need to unify physics. The experimenters were working with high energy machines and continually discovering new particles, which all had a claim to be considered elementary. Heisenberg proposed that they should all correspond to solutions of a single equation. This would be a wonderful idea if it would work. Heisenberg set up a certain cubic equation which he

hoped would do the trick and worked on it for many years. But he met formidable difficulties and had only limited success. So the value of this equation is doubtful.

What is the status of Heisenberg's discoveries at the present time? His mechanics has had tremendous success in all applications except those involving very high energies or very small distances. It provides the basis for describing all the ordinary processes of physics and chemistry. However, in the domain of very high energies or very small distances, the limitations of Heisenberg's theory show up. One can no longer make calculations with confidence.

Even here, however, the general Heisenberg idea of non-commutative algebra reigns. For describing the internal degrees of freedom of the elementary particles one needs non-commuting dynamical variables. On multiplying these together one gets mathematical groups. The evidence which is being provided by the big accelerators tells us what the groups are. A knowledge of the groups leads to a classicifation of the particles and tells us what quantum numbers are needed to describe the internal motion.

In looking back over the course of Heisenberg's discoveries one can see a lesson for present-day physicists. Probably the change that is now needed is comparable to the passage from Bohr's orbit theory to Heisenberg's matrix mechanics. Possibly it is just about as fundamental as the introduction of non-commutative algebra. If this is the case, we can hardly hope that it will be brought about by any direct attack. Some kind of circuitous route will be needed, like that which Heisenberg pioneered, which may well serve as a model for the future.