PONTIFICIA
ACADEMIA
SCIENTIARVM

COMMENTARII

Vol. II N. 39

K. E. BULLEN

THE SOLIDITY OF THE EARTH'S INNER CORE

EX AEDIBVS ACADEMICIS IN CIVITATE VATICANA

PONTIFICIA
ACADEMIA
SCIENTIARVM

COMMENTARII
Vol. II - N. 39
pag. 1-16

THE SOLIDITY OF THE EARTH'S INNER CORE

K. E. BULLEN
Pontifical Academician

SVMMARIVM — Auctor soliditatis nuclei interioris orbis terrarum perscrutatur. Inter alia revocatur doctrina Auctoris, quod pertinet ad compressionem et compressibilitatem. Respiciuntur insuper observationes recentes de terrestris oscillationibus deque undis sismicis *PKJKP*.

1. *Introduction*

During the past two years, the inference that the Earth's inner core is solid has received strong confirmation. The present paper summarizes the chief lines of evidence leading to this inference.

The interior of the Earth includes three main regions: the "shell" or "mantle", extending (approximately) to depth z = 2,900 km; the "outer core" (2,900 $< z <$ 5,150 km); and the inner core, of radius about 1,200 km. The existence of a core was established in seismic observations of OLDHAM (1906). The thickness of the mantle was indicated as near 2,900 km by GUTENBERG in 1914, with close confirmation in a fine

Paper presented on April 14th, 1972, during the Plenary Session of the Pontifical Academy of Sciences.

analysis by JEFFREYS (1939). Evidence that the core consists of an outer and an inner part was put forward by LEHMANN (1936) and confirmed in work of GUTENBERG and RICHTER (1938) and JEFFREYS (1939).

The Earth, being a deformable medium, transmits bodily through its interior two types of waves, P and S. The speeds α and β of P and S waves at any point of the interior are given to good reliability (see e.g. BULLEN, 1965a) by

$$(1) \qquad \alpha^2 \rho = k + 4\mu/3,$$

$$(2) \qquad \beta^2 \rho = \mu\,,$$

where ρ, k and μ denote the density, incompressibility and rigidity at the point. It will be convenient to introduce Φ, where

$$(3) \qquad \Phi = k/\rho = \alpha^2 - 4\beta^2/3.$$

In deriving (1) and (2), the Earth is treated as perfectly elastic and isotropic, and linear strain theory is applied in treating the strains which, during the wave motion, are superposed on the strains already present in the Earth's equilibrium configuration. The procedure is of course justified to the extent that its theoretical consequences agree with the relevant observations. For all ordinary problems of seismology, the observations have shown the procedure to be closely reliable. In particular, it is appropriate to use stress-strain relations for the entire interior of the Earth which contain just two elasticity parameters; these parameters may be, and most appropriately are, taken as k and μ. As with all relations in science, the stress-strain relations have the character of a mathematical model.

In contexts where stress-strain relations involving just k and μ give a suitable representation, it is appropriate to term a material as solid or fluid according as μ/k is significant (in practice of order at least 0.01) or not. For the ordinary problems of seismic wave transmission, this particular model serves well and the terms solid and fluid therefore have meaning throughout the entire Earth, in the sense defined. The terms solid and fluid are to be interpreted in this way throughout this paper. (In some other geophysical contexts, including some special seismological contexts, the two-parameter model is not adequate; the terms solid and fluid then need further elaboration).

The mantle of the Earth has, since around 1910, been known to be solid in the sense defined, because it transmits S as well as P waves throughout. But no S waves have been detected below the mantle until very recently, and then not in the outer core. Direct evidence indicating that the outer core is fluid was supplied by JEFFREYS in 1926, using studies of solid tides of the Earth; quantitative detail was supplied by TAKEUCHI (1950) and MOLODENSKI (1955).

Following the discovery of the inner core, it was tacitly assumed for some years that the inner core would be fluid, like the outer core. (In 1940, BIRCH mentioned the possibility that the inner core might consist of a solid modification of iron, but did not discuss the possibility quantitatively). As will be seen in the following Sections, a large body of evidence now points strongly towards rigidity.

2. *The* k-p *theory*

Over the period 1936-42, the writer worked out a density distribution for the Earth. By virtue of (1) and (2), the distribution carried with it distributions of k and μ in the mantle and outer core (assuming $\mu = 0$ in the outer core).

Distributions of the pressure p and gravitational intensity g were also yielded. (It is sufficient for the present discussion to represent the internal stress in the equilibrium state in terms of the single parameter p).

An arresting feature of the distributions was that, whereas a large jump in ρ (from the order of 5½ to nearly 10 g/cm^3) was indicated at the mantle-core boundary N, and whereas μ was indicated as falling suddenly at N from 3×10^{12} dyn/cm^2 on the mantle side to a negligible value on the core side, both k and dk/dp were indicated as nearly continuous at N. This feature, along with the limited experimental evidence available at the time on the variation of k with p for various materials over the range $0 < p < 10^{11}$ dyn/cm^2 (at N, p $\backsimeq$ 1.35×10^{12} dyn/cm^2), emboldened the writer in 1946 to propose, initially as a trial hypothesis, that k and dk/dp are smoothly varying functions of p throughout the whole Earth below the level ($z \backsimeq 1{,}000$ km) at which $p = 0.4 \times 10^{12}$ dyn/cm^2, to a large extent independently of chemical composition. The theory that this is approximately the case will be referred to as the k-p theory.

The hypothesis carried with it several interesting implications, including the implication that the Earth's inner core is solid. Since 1946, a variety of new observational data has required the original (1936-42) distributions of ρ, k, μ, p and g to be revised, the most important revision arising from an unexpectedly large reduction (see COOK, 1963) from 0.3336 to 0.3309 in the estimated moment of inertia coefficient of the Earth, following analyses of artificial satellite orbits. But the near-continuity of k and dk/dp at N have been fully sustained; if anything, the new data have strengthened the case for this near-continuity at N. (For further details on the k-p hypothesis, see BULLEN, 1949, 1950a, 1953, 1958, 1968a, 1972b).

Stripped to its essence, the original argument for the solidity of the inner core is as follows. An essential part of LEHMANN'S discovery is that α jumps by about 10 per cent across the boundary, L say, between the outer and inner core. (Complications connected with the possible presence of transition layers between the outer and inner core proper are being ignored in the present brief summary). Suppose that the inner core were fluid, i.e. $\mu = 0$. Then the 20 per cent jump in α^2 would, by (1), require either that ρ diminishes sharply at L, which is extremely improbable, or that k increases so rapidly as to be discordant with the k-p theory. Thus the k-p theory and LEHMANN'S finding on α together entail that the inner core must have significant rigidity. Plausibility is added by the fact that any jump in k at L is expected to be much smaller than the jump at N since most other physical properties suffer much smaller changes at L than at N.

3. *The seismic phase PKJKP*

Since a solid, but not a fluid, transmits seismic S waves, detection of S waves in the inner core would provide direct evidence of solidity. Theoretical travel times and amplitudes for a seismic phase *PKJKP* were calculated (BULLEN, 1950b, 1951) with a view to possibly obtaining such evidence. The phase *PKJKP* corresponds to a wave with a segment in P type descending from an earthquake focus through the mantle, refracted at N into a P wave (symbolized as K) descending through the outer core, refracted at L into an S wave segment (symbolized as J) passing through the inner core, refracted upward at L into P type and again at N into P type, and continuing upward to be recorded at the Earth's outer surface.

The amplitude calculations showed that, with the seismological equipment available at the time, the phase *PKJKP* was, at best, on the border of observability, and then only on assuming the most favourable type of boundary conditions at L. During the immediately following years, several seismologists offered evidence of detection of *PKJKP*, but the writer after statistical scrutiny was not convinced that the phase had been authentically observed (see e.g. BULLEN, 1956).

The original calculations for *PKJKP* had assumed $\beta \backsimeq$ 4-5 km/sec in the inner core. This value was later reduced (BULLEN, 1965b), and the model B_2 of BULLEN and HADDON (1967) had β ranging from 3.8 at the top of the inner core to 2.9 km/sec at the centre.

4. *Supporting evidence on solidity of the inner core*

In the years following 1946, evidence from a variety of sources supported, with varying degrees of strength, the notion that the inner core is solid. So far as the writer is aware, there is no evidence arguing to the contrary. Till quite recently, however, the supporting evidence fell short of establishing the solidity directly. Following are some examples of the earlier supporting evidence.

SIMON (1953) inferred that a transition from liquid to solid iron at L at the pressure involved (about 3 million atmospheres) would correspond to a temperature of 3600°C, a value that agrees well with other evidence. JACOBS (1954) pointed out that an Earth wholly molten at some earlier stage could well have started solidifying from the centre upward, a solid inner core growing until the stage was reached when the curve representing the adiabatic temperature distribution in the Earth intersected the melting-point curve at two points corresponding to L and N, leaving a molten outer core trapped in between. LUBIMOVA (1956) independently inferred from thermodynamical calculations that the inner core is below melting point.

CHANDRASEKHAR (1952) calculated that a boundary such as a solid inner core would provide is necessary to sustain the modes of convection in the fluid outer core required on the currently favoured theory of the Earth's main magnetic field.

Data from theoretical physics on the variation of k with atomic number at extreme pressures, along with observational evidence at the greatest pressures obtained in laboratories at the time, led to an estimate (Bullen, 1952) that any sharp increase in k at L is unlikely to exceed one-fifth of the increase that would implied by the seismological data on α if the inner core were fluid.

Caloi (1961) detected a seismic phase *PKiKP* (the *i* indicating upward reflection at the inner core boundary) and thence inferred supporting evidence for inner core solidity.

An adaptation by Birch (1952) of Murnaghan's finite-strain theory provided independent evidence on the variation of k and dk/dp with p. Leading formulae in Birch's theory are

$$p = 3k_0 f\,(1 + 2f)^{5/2}, \tag{4}$$

$$k = k_0\,(1 + 2f)^{5/2}\,(1 + 7f), \tag{5}$$

$$3dk/dp = 7 + 5\,(1 + 7f)^{-1}, \tag{6}$$

where f denotes the compression and the zero subscript relates to zero pressure. The equations (4)-(6) relate to a material of constant chemical composition. (Minor thermodynamical complications are here being ignored). Birch estimated f in the mantle by a rather complicated process. Using the simple relation

$$f = p/(3k - 7p), \tag{7}$$

derivable from (4) and (5), BULLEN (1968b) estimated f throughout the whole Earth. Neglecting possible large phase changes inside the Earth, the preferred value of f is about 0.13 immediately above and below N and reaches 0.19 at the Earth's centre. Knowing f, values of k and dk/dp entailed by BIRCH's theory can then be derived using (4)-(6). The results, along with various other deductive consequences (BULLEN, 1968c, 1969), have, within the uncertainties of BIRCH's theory (see BULLEN 1970, 1972b; THOMSEN and ANDERSON, 1969; THOMSEN, 1970), confirmed the general reliability of the values of k and dk/dp originally used in formulating the k-p theory.

5. *Evidence from shock-wave experiments*

After assembling evidence from shock-wave experiments on a variety of materials at pressures, of the order $3\text{-}4 \times 10^{12}$ dyn/cm^2, reached inside the inner core, BIRCH (1961) inferred that the Earth's central density ρ' is not likely to be much greater than 13 g/cm^3.

The WILLIAMSON-ADAMS equation (1923, 1925) for density gradient (derived using $dp/dz = g\rho$ and $\Phi = k/\rho = dp/d\rho$), namely,

$$(8) \qquad d\rho/dz = g\rho/\Phi \ ,$$

is now superseded (BULLEN, 1963, 1967) by

$$(9) \qquad d\rho/dz = \eta g\rho/\Phi \ ,$$

where

$$(10) \qquad \eta = dk/dp - g^{-1}d\Phi/dz \ .$$

The equation (8), through its dependence on the formula $k = \rho dp/d\rho$, is restricted to the case of chemical homogeneity and adiabatic temperature gradient. Although quite useful as a first approximation in some parts of the Earth, (8) is seriously unreliable in other parts (BULLEN, 1936). The relation (10) has much utility through the fact that dk/dp and g can now be assessed within close limits inside most of the Earth, while $d\Phi/dz$ may be estimated from seismic data on α and β.

The value of ρ' being at least 12.3 g/cm³ (BULLEN, 1936), it can be shown (BULLEN, 1965b) that the assumption of BIRCH's value 13 g/cm³ for ρ' entails that $\eta \backsimeq 1$ throughout the core (except possibly inside limited transition zones). The evidence is strong that the value of dk/dp is at least 3 in the inner core. The available seismic evidence gives $d\alpha/dz \backsimeq 0$ in the inner core. Assuming these numerical results, (10) gives

$$(11) \qquad 1 \backsimeq 3 + g^{-1}d(4\beta^2/3)/dz\,,$$

which requires $d\beta/dz$ to be significantly negative (BULLEN, 1964) inside the inner core. On proceeding to further detail, $d\mu/dz$ is also indicated as negative. This in turn is evidence that the inner core is solid since a region cannot be entirely fluid if the rigidity is significantly changing inside it.

Thus the assumptions that ρ' does not much exceed 13 g/cm³ and that $d\alpha/dz \backsimeq 0$ in the inner core together supply additional evidence that the inner core is solid.

The theory involved in the above argument appears unshakable, but some uncertainty attaches to the conclusion because of uncertainty in the observational data. Possible revision of the data on $d\alpha/dz$ could annul this line of evidence. A further calculation (BULLEN, 1965b) showed that the evidence would also be annulled should ρ' exceed 14.7 g/cm³. BOLT

(1970) inferred from other seismological evidence, however, that $\rho' \leqslant 14$ g/cm^3.

The above illustrates arguments that have been brought to bear in seeking to establish the solidity of the inner core. They all act in the direction of increasing the probability that the inner core is solid, but fall short of making a direct case.

6. *Recent evidence*

6.1. *Attenuation of P waves inside the Earth's core*

Sacks (1970), in a spectral analysis of P wave data, estimated the attenuation parameter Q (for definition of Q, see Jeffreys, 1970, p. 333) to be of order 200 in the outermost part of the inner core, rising rapidly to the order of 600 immediately below. In the fluid outer core, Q is an order of magnitude higher. These results led Sacks to suggest that the top of the inner core may be in a state of partial melting which is a prelude to the setting in of rigidity lower down in the inner core. He also pointed out that this state of affairs would accentuate the difficulty of detecting S waves in the inner core while at the same time providing new evidence that there is rigidity.

6.2. *Evidence from free Earth oscillation data*

The indisputable recording of free Earth oscillations following the Chilean earthquake of 1960 May 22 added important new evidence on the distributions of ρ, k and μ inside the Earth. The periods of considerably more than 100 modes of free Earth oscillations have now been measured to fairly good precision, and each observed period adds one new numerical condition which the distributions must satisfy.

An analysis of the new data made soon after the Chilean earthquake confirmed the reliability of the 1936-42 distributions of ρ, k and μ within the previously estimated accuracy. For example, the density distribution was confirmed as accurate within 5 per cent or less for $0 < z < 5{,}000$ km, approx. Subsequently, the implications of free Earth oscillation data on the determination of the internal structure of the Earth have been examined in considerable detail.

Using free Earth oscillation data, DERR (1969) inferred that the inner core must be solid, but it later became necessary to re-interpret part of his data. Work of BULLEN and HADDON (see e.g. papers of 1967, 1969, 1970) indicated that, on the data available by 1969, one could not in this way fully establish the solidity of the inner core. HADDON (see BULLEN, 1972a), concluded that the data available by 1969 did, however, indicate the presence of significant rigidity somewhere inside the core; of course, on other grounds, the location would be expected to be the inner core. More recently, DZIEWONSKI and GILBERT (1971) produced a large quantity of new observational free Earth oscillation data, including data on the periods of a number of pertinent overtones. With the addition of these additional data, the case for a solid inner core has become fairly convincing. The calculations of DZIEWONSKI and GILBERT yielded $\beta \approx 3.5$ km/sec in the inner core. This would entail that μ is at least 1.5×10^{12} dyn/cm^2.

6.3. *Observations of the phase PKJKP*

With the advent of modern seismic array stations and filtering techniques, the chances of detecting the phase *PKJKP* on seismic records have been much enhanced. JULIAN, DAVIES and SHEPPARD (1972) now claim to have detected the phase, using these techniques. Their evidence is rather limited, but they have applied various tests carefully in making their interpretations. They infer that $\beta = 2.95$ km/sec. This value would entail an inner-core rigidity not less than 1.0×10^{12} dyn/cm^2, which may be compared with the values 0.6 and 3.0×10^{12} dyn/cm^2 at the top and bottom of the mantle. If further observations confirm the work of these investigators, the solidity of the inner core can be regarded as directly established.

As it is, the overall case is already quite strong.

The manuscript of this paper was typed at the International Institute of Seismology and Earthquake Engineering, Tokyo, and the writer wishes to express his thanks for the services kindly made available.

University of Sydney

REFERENCES

ADAMS, L.H. and WILLIAMSON, E.D., *The composition of the Earth's interior.* « Smithsonian Rept. 1923 », pp. 241-260 (1925).

BIRCH, F., *The alpha-gamma transformation of iron at high pressures, and the problem of the Earth's magnetism.* « Amer. J. Sci. », *238*, 192-211 (1940).

— *Elasticity and constitution of the Earth's interior.* « J. Geophys. Res. », *57*, 227-286 (1952).

— *Composition of the Earth's mantle.* « Geophys. J., R. Astr. Soc. », *4*, 295-311 (1961).

BOLT, B.A., *Upper bound to the density jump at the boundary of the Earth's inner core.* « Nature, Lond. », *228*, 148-150 (1970).

BULLEN, K.E., *The variation of density and the ellipticities of strata of equal density within the Earth.* « Mon. Not. R. Astr. Soc., Geophys. Suppl. », *3*, 395-401 (1936).

— *The problem of the Earth's density variation.* « Bull. Seismol. Soc. Amer. », *30*, 235-250 (1940).

— *The density variation of the Earth's central core.* « Bull. Seismol. Soc. Amer. », *32*, 19-29 (1942).

— *A hypothesis on compressibility at pressures of the order of a million atmospheres.* « Nature, Lond. », *157*, 405 (1946).

— *Compressibility-pressure hypothesis and the Earth's interior.* « Mon. Not. R. Astr. Soc., Geophys. Suppl. », *5*, 355-368 (1949).

— *An Earth model based on a compressibility-pressure hypothesis.* « Mon. Not. R. Astr. Soc., Geophys. Suppl. », *6*, 50-59 (1950a).

— *Theoretical travel times of S waves in the Earth's inner core.* « Mon. Not. R. Astr. Soc., Geophys. Suppl. », *6*, 125-128 (1950b).

— *Theoretical amplitudes of the seismic phase* PKJKP. « Mon. Not. R. Astr. Soc., Geophys. Suppl. », *6*, 163-167 (1951).

— *On density and compressibility at pressures up to thirty million atmospheres.* « Mon. Not. R. Astr. Soc., Geophys. Suppl. », *6*, 383-401 (1952).

— *The rigidity of the Earth's inner core.* « Ann. Geofis. », *6*, 1-10 (1953).

— *Physical properties of the Earth's core.* « Ann. Géophys. », *11*, 53-64, (1955).

— *Note on the phase* PKJKP. « Bull. Seismol. Soc. Amer. », *46*, 333-334 (1956).

— *Solidity of the inner core.* In « Contributions in Geophysics ». (London: Pergamon), pp. 113-120 (1958).

— *An index of degree of chemical inhomogeneity in the Earth.* « Geophys. J., R. Astr. Soc. », 7, 584-592 (1963).

— *New evidence on rigidity in the Earth's core.* « Proc. Nat. Acad. Sci., Wash. », *52*, 38-42 (1964).

— *An Introduction to the Theory of Seismology.* (Cambridge University Press), 3rd ed. (1965a).

— *Models for the density and elasticity of the Earth's core.* « Geophys. J., R. Astr. Soc. », *9*, 233-252 (1965b).

— *Note on the coefficient* η. « Geophys. J., R. Astr. Soc. », *13*, 459 (1967).

— *Incompressibility at the Earth's mantle-core boundary.* « Proc. Nat. Acad. Sci., Wash. », *60*, 752-757 (1968a).

— *Compression in the Earth.* « Geophys. J., R. Astr. Soc. », *16*, 31-36 (1968b).

— *Dependence of compressibility and compression on chemical composition in finite-strain theory.* « Phys. Earth Planet. Interiors », *1*, 297-301 (1968c).

— *Seismic and related evidence on compressibility in the Earth.* In « The Application of Modern Physics to the Earth and Planetary Interiors ». (London: Wiley-Interscience), pp. 287-297 (1969).

— *Comparison of sources of evidence on the variation of incompressibility in the Earth's deeper interior.* « Phys. Earth Planet. Interiors », *3*, 36-40 (1970).

— *Some problems connected with constructing Earth models.* (In course of publication « Proc. Conf. on Solid Earth Problems », Buenos Aires 1970) (1972a).

— *Compressibility and planetary interiors.* (In course of publication « Phys. Earth Planet. Interiors ». (1972b).

Bullen, K.E. and Haddon, R.A., *Earth models based on compressibility theory.* « Phys. Earth Planet. Interiors », *1*, 1-13 (1967).

— *Evidence from seismology and related sources on the Earth's present internal structure.* « Phys. Earth Planet. Interiors », *2*, 342-349 (1970).

Caloi, P., *Seismic waves from the outer and inner core.* « Geophys. J., R. Astr. Soc. », *4*, 139-150 (1961).

Chandrasekhar, S., « Phil. Mag. », 7, 1317-1329 (1952).

Cook, A.H., *The contributions of satellites to the determination of the Earth's gravitational potential.* « Space Sci. Reviews », *2*, 355-437 (1963).

DERR, J.S., *Internal structure of the Earth inferred from free oscillations.* « J. Geophys. Res. », *74*, 5202-5220 (1969).

DZIEWONSKI, A.M. and GILBERT, F., *Solidity of the inner core inferred from normal mode observations.* « Nature, Lond. », *234*, 465-466 (1971).

GUTENBERG, B., *Über Erdbebenwellen, VIIA.* « Nachr. Ges. Wiss. Göttingen, Math.-Phys. Klasse », pp. 1-52 and 125-176 (1914).

GUTENBERG, B. and RICHTER, C.F., *P' and the Earth's core.* « Mon. Not. R. Astr. Soc., Geophys. Suppl. », *4*, 363-372 (1938).

HADDON, R.A. and BULLEN, K.E., *An Earth model incorporating free Earth oscillation data.* « Phys. Earth Planet. Interiors », *2*, 35-49 (1969).

JACOBS, J.A., *Temperature distribution within the Earth's core.* « Nature, Lond. », *173*, 258 (1954).

JEFFREYS, H., *The rigidity of the Earth's central core.* « Mon. Not. R. Astr. Soc., Geophys. Suppl. », *1*, 371-383 (1926).

— *The times of the core waves.* « Mon. Not. R. Astr. Soc., Geophys. Suppl. », *4*, 548-561 and 594-615 (1939).

JEFFREYS, SIR H., *The Earth.* (Cambridge University Press), 5th ed. (1970).

JULIAN, B.R., DAVIES, D. and SHEPPARD, R.M., *PKJKP.* « Nature, Lond. », *235*, 317-318 (1972).

LEHMANN, INGE, *P'.* « Publ. Bur. Centr. Séism. Internat. A », *14*, 3-31 (1936).

LUBIMOVA, H.A., *Thermal history of the Earth and its geophysical effect.* « Dokl. Akad. Nauk, U.S.S.R. », *107* (11), 55-58 (1956).

MOLODENSKI, M.S., *Density and elasticity within the Earth.* « Trans. Geophys. Inst. Acad. Sci., U.S.S.R. », vol. 26. (1955).

OLDHAM, R.D., *Constitution of the Earth as revealed by earthquakes.* « Quart. J. Geol. Soc. », *62*, 456-475 (1906).

SACKS, I.S., *Personal communication.* (1970).

SIMON, LORD (F.E.), *The melting point of iron at high pressures.* « Nature, Lond. », *172*, 746 (1953).

TAKEUCHI, H., *On the earth tide in the compressible Earth of varying density and elasticity.* « Trans. Amer. Geophys. Un. », *31*, 651-689 (1950).

THOMSEN, L., *On the fourth-order anharmonic equation of state of solids.* « J. Phys. Chem. Solids », pp 2003-2016 (1970).

THOMSEN, L. and ANDERSON, O.L., *On the high-temperature equation of state of solids.* « J. Geophys. Res. », *74*, 981-991 (1969).

WILLIAMSON, E.D. and ADAMS, L.H., *Density distribution in the Earth.* « J. Wash. Acad. Sci. », *13*, 413-428 (1923).