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The Emergence of Order

Walter Thirring

Explanation

We study the mutation-selection dynamics. The dynamical variables are the p_i. They are positive and their sum is normalized to unity. The α_{ik} are parameters which determin the evolution. The p_i can be thought of probabilities of populations. Their randomness is given by the entropy S. The α reflects the accidental situation in which the system is embedded. The main question is whether they will lead to order (low S) or chaos (high S)

$$\frac{dp_i}{dt} = \sum_k \alpha_{ik} p_k - p_i \sum_{j,k} \alpha_{jk} p_k$$

$$i, j, k = 1, 2, \ldots, d$$

$$\sum_i p_i = 1$$

$$S = \sum_k p_k (1 - p_k)$$

It turns out that the p_i generically tend to a limit which is independent of their initial values and are determined by the α_{ik}. As first orientation we do not restrain the α and let them be random numbers within certain bounds. This leads to a distribution of the final entropies which depends only on the dimension d of the p_i space, ($i=1\ldots d$).

The next figure shows this distribution and we see that for each d the entropy clusters around the maximal value. There is no creation of order from disorder. Next we consider a hierarchical structure of the p_i so that the α tends to a triangular matrix. In this case order is created out of disorder.

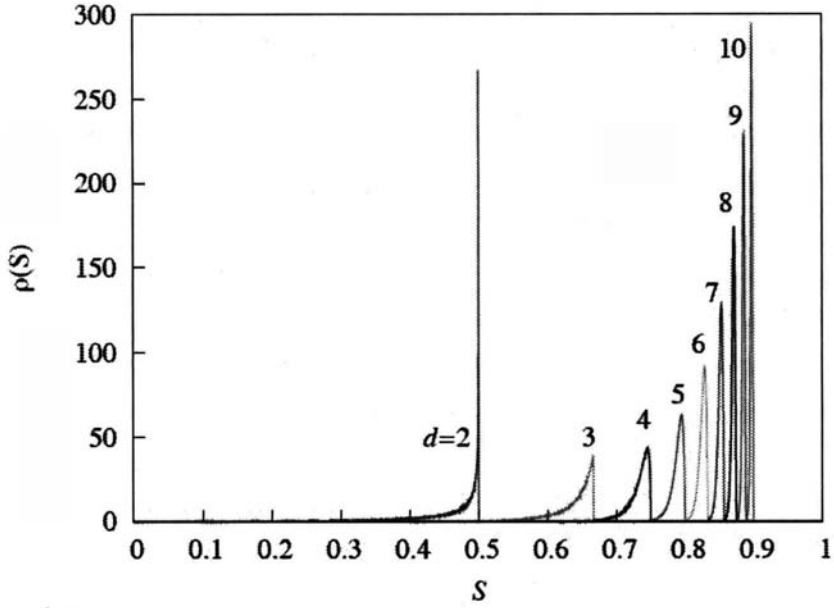
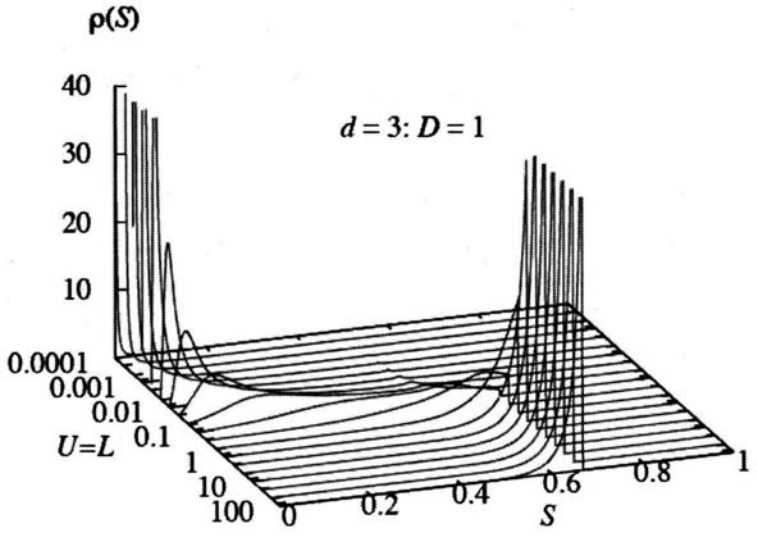


Figure 1.



Die Entropie als Funktion vom Verhältnis der Nichtdiogonalelemente U, L zu den Diagonalelementen D. Die Schranken für erstere wurden gleich gewählt.

Figure 2.

Such a behaviour can be easily understood by interpreting the p_i financially of the assets of d players. The dynamically equation solves the payments of player i to player k and the order of the indices i reflect the richness of player i. Triangularity of the α means that the payments mainly go from the poor to the rich. Therefore eventually the latter end up having all the money. It is interesting to see that this situation changes already if the non-triangularity reaches a few percent.