Paths of Discovery
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GEOMAGNETISM, 'VACILLATION', ATMOSPHERIC PREDICTABILITY AND 'DETERMINISTIC CHAOS'

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1. Prologue

'Discoveries' – as the organisers of this symposium on 'Paths of Discovery' emphasise – 'are at the basis of new knowledge'. Some discoveries are made upon verification or 'falsification' of a theory, but in many cases serendipity plays a key rôle. Then a discovery is made whilst something else is being sought but the scientific mind and intuition of the researcher become directed towards the unexpected.

Serendipity certainly featured in some of the main events outlined in this contribution to the symposium. They started in 1947 when P.M.S. Blackett, a cosmic ray physicist then at the University of Manchester, proposed a testable new theory of the Earth's magnetism [1], which over the next few years he and colleagues succeeded in 'falsifying' by observation and experiment. The events ended in 1963 when E.N. Lorenz, a dynamical meteorologist at the Massachusetts Institute of Technology (MIT), published an account of his work on deterministic non-periodic fluid flow, motivated by his interest in the predictability of weather patterns. Lorenz's paper [2] was later recognised by scientists in other disciplines and by mathematicians as a seminal contribution to what subsequently became known as 'chaos theory'. This now influences ideas and methodologies in many branches of science and technology.

Linking these studies were quantitative laboratory experiments in which I discovered in spinning fluids subject to steady (thermal) forcing several nonlinear régimes of flow of varying degrees of complexity in their spatial and temporal characteristics, including (multiply-)periodic ('vacillation') and highly aperiodic ('chaotic') régimes. Undertaken from 1950 to 1953 at the University of Cambridge (and later repeated and their findings confirmed by D. Fultz and his colleagues at the University of Chicago), the experiments were motivated in the first instance by my interest in geomagnetism and motions in the metallic core of the Earth. But they were to attract the attention of meteorologists engaged in research on large-scale atmospheric motions and influence Lorenz's mathematical work on atmospheric predictability and nonlinear dynamical systems.

The present article is based on notes prepared originally in response to interest expressed by mathematicians and others in the geophysical background to the experiments.

2. Geomagnetism and Motions in the Earth's Liquid Outer Core

Speculations as to the origin of the Earth's magnetism go back several centuries, but geophysicists now agree that the phenomenon must be due to ordinary electric currents flowing within the Earth's metallic core, where they experience least resistance. Chemical and thermoelectric effects are unlikely to be strong enough to account for the electromotive forces needed to maintain the currents against ohmic dissipation, but motional induction involving hydrodynamical flow in the liquid outer core cannot be ruled out on quantitative grounds. This is the main reason why theoretical geophysicists – now equipped with powerful super-computers – have since the mid-1940s been prepared to wrestle with the mathematical complexities of 'self-exciting dynamos' in electrically-conducting fluids.

Dynamos convert the kinetic energy associated with the motion of an electrical conductor through a magnetic field into the magnetic energy associated with the electric currents thus generated in the moving conductor by the process of motional induction. In self-exciting dynamos permanent magnets are not necessarily involved; all that is needed is the presence of a very weak background magnetic field when the dynamo is started up. The self-excitation principle was discovered in the 1860s by engineers concerned with the development of practical systems of public electricity supply [3], who experimented with devices in which the rotating armature was connected by sliding electrical contacts to a suitably-oriented stationary field coil.

Such devices are topologically more complex in their structure than a continuous body of electrically-conducting fluid, such as the Earth's liquid metallic outer core. So it is by no means obvious that self-exciting dynamo action is possible in fluid systems, but it turns out to be true. From the equations of electrodynamics, theoreticians seeking mathematical 'existence theorems' have been able to show that in an electrically-conducting fluid most flows of sufficient rapidity and complexity in form are able to produce and maintain a magnetic field against ohmic dissipation. The first existence theorems [4] were produced, independently, by G.E. Backus and A. Herzenberg in the same year, 1958. This was nearly four decades after J. Larmor, in a paper on solar magnetism, had made the original suggestion that self-exciting dynamo action might be possible in a moving fluid [5]. His important idea appeared less attractive in 1934, when T.G. Cowling [6] showed that motional induction was incapable of maintaining magnetic fields of the limited class that possess an axis of symmetry. According to an aggrieved Larmor [7], even the Council of the Royal Astronomical Society were prompted by Cowling's *non*-existence theorem to respond negatively to his idea. Larmor saw on quantitative grounds that some kind of dynamo mechanism was needed to explain solar magnetism.

It was against this background of uncertainty that Blackett [1] in 1947 offered geophysicists a new theory of the origin of the Earth's main magnetic field. Significantly, this was done several years before rock magnetism studies had produced convincing evidence of polarity reversals of the field. Blackett was then the Head of the University of Manchester's large and lively Department of Physics (and due to receive a Nobel prize in the following year for his work on cosmic rays). According to his theory, which invoked an earlier suggestion associated with the names of H.A. Wilson and E. Schrödinger, [1, 8] the main magnetic fields of the Earth, Sun and any other rotating astronomical body were all manifestations of a new law of Nature, whereby any massive rotating body would be magnetic in virtue of its rotation. Its magnetic moment would be proportional to its spin angular momentum with the constant of proportionality equal to the square root of the universal gravitational constant divided by twice the speed of light, implying that if correct the theory would provide a basis for unifying the laws of gravity and electromagnetism.

E.C. Bullard quickly pointed out that the new theory could be tested by determining the vertical variation of the geomagnetic field in the upper reaches of the Earth. Blackett responded by setting up a research team under one of his staff members, S.K. Runcorn, charged with the task of measuring the field in deep coal mines. The theory was soon 'falsified' by the findings of the 'mine experiment' [9] and also by a direct laboratory experiment carried out by Blackett himself [10].

Geophysicists concerned with the origin of the main geomagnetic field were thus left with little choice but to confront the mathematical complexities of 'geodynamo' theory. These stem from the essential nonlinearity of the equations of magnetohydrodynamics (MHD) that govern flows in electrically-conducting fluids, a subject then in its infancy associated with the name of H. Alfvén. MHD phenomena such as self-exciting fluid dynamos abound in large-scale systems such as stars and planets, where typical values of the 'magnetic Reynolds number' $R=UL\mu\sigma$ can be high. (Here U is a characteristic flow speed, L a characteristic length scale, μ the magnetic permeability of the fluid and σ its electrical conductivity). But the scope for investigating such phenomena on the small scale of the terrestrial laboratory is very limited, owing to the difficulty with available conducting fluids of attaining high values of R.

Buoyancy forces due to the action of gravity on density inhomogeneities associated with differential heating and cooling produce fluid motions in stars and planets. The motions transfer heat by (free) thermal convection and their patterns are influenced by gyroscopic (Coriolis) forces due to general rotation. W.M. Elsasser pointed out in 1939 that the influence of Coriolis forces on convective motions in the Earth's liquid outer core may somehow account for the approximate alignment of the geomagnetic field with the Earth's rotation axis [11] – which for nearly a thousand years has been exploited by navigators using the magnetic compass.

3. 'Vacillation' and Other Régimes of Thermal Convection in a Rotating Liquid 'Annulus'

In 1948, as an impecunious undergraduate studying physics at the University of Manchester needing part-time paid employment, I joined the 'mine experiment' team as an assistant. The experience of working with Runcorn and his team stimulated my interest in geomagnetism and introduced me to the literature of the subject. Encouraged by Blackett and Runcorn, on graduating in 1950 I enrolled as a PhD student in the small Department of Geodesy and Geophysics at the University of Cambridge, where research in geodesy and seismology was already well established and new (and highly fruitful) initiatives were being taken in other areas – in marine geophysics by M.N. Hill and in palaeomagnetism by J. Hospers and Runcorn (who had moved from Manchester to Cambridge).

With some experience in experimental physics (but none in fluid dynamics), on reaching Cambridge I started some laboratory experiments on thermal convection in a cylindrical annulus of liquid (water) spinning about a vertical axis and subjected to an impressed axisymmetric horizontal temperature gradient. The necessary apparatus was quickly designed and constructed using equipment and other resources available in the department, including a war-surplus synchronous electric motor, a steel turntable used previously for grinding rocks, a supply of brass and glass tubing up to about 10 cm. in diameter and a recording camera incorporating a set of galvanometers which was no longer needed for field work in seismology. The resources also included, crucially, the facilities of a small workshop where research students could design and construct apparatus under the guidance of an experienced technician, L. Flavell.

My initial motivation amounted to nothing more than the hope that laboratory work on buoyancy-driven flows influenced by Coriolis forces due to general rotation might somehow shed light on motions in the Earth's liquid outer core. Luckily, promising lines of investigation emerged as soon as the apparatus was run for the first time, when a persistent regular flow pattern of four waves marked out by a meandering jet stream was seen at the top surface of the convecting liquid. By increasing the value of the steady angular speed of rotation of the apparatus, Ω (say), it was possible to increase the number of regular waves, M, but not beyond a point at which the pattern became highly irregular ('chaotic'). M could be decreased by reducing Ω, but not beyond a point at which the non-axisymmetric (N-) flow gave way to axisymmetric (A-) flow (see Figure 1 below).

The next steps were to investigate systematically how this behaviour depended not only on Ω but also on other impressed experimental conditions, namely the fractional density contrast ($\Delta\rho/\rho$) associated with the temperature difference maintained between the cylindrical side-walls of the 'annular' convection chamber, the depth (d) of the liquid within the 'annulus', and the width (b-a) of the gap between the side-walls – keeping the radius of curvature, b, of the outer side-wall fixed in the first instance. Empirical criteria were thus deduced for the occurrence of transitions (a) between the A-régime and the regular non-axisymmetric (RN-) régime, and (b) between the RN-régime and the irregular non-axisymmetric (IN-) régime.

The first of these transitions, (a), was found to occur at a critical value of the dimensionless parameter

$$\Theta = [gd\Delta\rho/\rho]/[\Omega^2(b\text{-}a)^2], \qquad (1)$$

where g denotes the acceleration due to gravity, which was typically much stronger than centripetal acceleration. The criterion indicates that loss of stability of the A-régime involves the conversion of potential energy into kinetic energy.

As to the criterion for the transition between the RN-régime and the IN-régime, the fully-developed regular waves of the RN-régime were found to be characterised by azimuthal wavelengths never exceeding approximately $3(b\text{-}a)/2$, with little dependence on d. The criterion implies a simple dependence of the value of M at the transition on the ratio $\Gamma=[b\text{-}a]/[(b+a)/2]$ and it indicates that the chaotic IN-régime ('geostrophic turbulence') arises when the RN-régime ('vacillation', see below) loses its stability through the non-linear transfer of kinetic energy between Fourier modes.

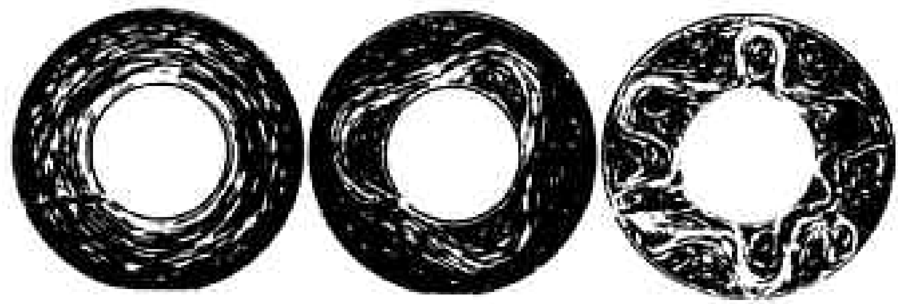

Figure 1. Streak photographs taken to illustrate three typical top-surface flow patterns, the first in the axisymmetic régime, the second in the regular non-axisymmetric régime (of 'vacillation') with M=3, and the third in the irregular ('chaotic') non-axisymmetric régime (of 'geostrophic turbulence'). The respective values of Ω were 0.34, 1.19 and 5.02 radians per second; other impressed conditions were held fixed.

Later experiments using glycerol/water mixtures indicated how these empirical criteria depend on the viscosity and thermal conductivity of the working liquid. The dependence is weak when Ω is so high that viscous effects are weak, but at low values of Ω the criteria exhibit dependence on the coefficient of viscosity, for which there is a critical value – found to depend on d, $(b\text{-}a)$ and Ω – below which axisymmetric flow occurs for all values of Θ.

The procedure followed in most investigations of the RN-régime involved setting Ω and other quantities required to specify the impressed experimental conditions at pre-determined values, and then waiting until transients had died away before measuring various properties of the flow that persisted. In some cases the persistent pattern of waves turned out to be steady (apart from a steady drift of the pattern relative to the annular convection chamber), but in others the pattern would undergo regular periodic fluctuations of various kinds. In the simplest of these the pattern exhibited pulsations in amplitude, which at their most pronounced were accompanied by alternations in the number of waves, M, from one cycle to the next. In other time-varying cases a sizeable local distortion of the wave pattern, sometimes amounting to the splitting of a wave, was seen to progress around the pattern, or the shape of the whole pattern would waver.

Significantly – in a manner reminiscent of the behaviour of a 'pin-ball' machine – when a number of experiments were carried out under the same impressed conditions there was a spread in values of M of the patterns that persisted, rather than a unique value of M. Thus, in a large number of trials under the conditions, say, of the second picture in Figure 1 (where M happens to be equal to 3), with each trial starting with the thorough stirring of the working liquid and then waiting for the resulting small-scale motions to die away, the resulting value of M of the persistent pattern that eventually formed would be equal to 2, 3 or 4, with relative probabilities depending on the value of the dimensionless parameter Θ.

Never expecting the term to stray beyond my laboratory notebook, I used 'vacillation' to denote the most extreme form of periodic 'wavering' seen in the experiments on the RN-régime. This occurred near the transition to the IN-régime. At one phase of the cycle the meandering jet stream gave way to separate eddies, which in turn decayed allowing the jet stream to reform, and so on. But when other workers took up annulus experiments (see below) some used the term 'vacillation' to signify *any* flow exhibiting regular periodic fluctuations. This made it necessary to introduce the terms 'shape vacillation', 'amplitude vacillation', 'wave-number vacillation', etc., leaving 'vacillation' on its own as an alternative term for the regular non-axisymmetric (RN-) regime (with *steady* non-axisymmetric flows as extreme cases when fluctuations are imperceptible).

Before leaving Cambridge in 1953 I completed my experimental work there by making – over substantial ranges of impressed conditions – further determinations of total convective heat transfer, flow velocities and patterns of temperature variations (using small arrays of thermocouples), and also of the non-unique dependence of M on Θ, etc., in the RN-régime. My main findings were summarised in two short papers [12], but several years (including a period of compulsory National Service) elapsed before any of the details of methods and results given in my PhD dissertation were submitted for publication in the open literature [13].

4. Geophysical and Astrophysical Fluid Dynamics and Dynamical Meteorology

General considerations of the dynamics of convective heat transfer in spinning fluids indicate that Coriolis forces promote departures from axial symmetry in systems characterised by axial symmetry in their boundary conditions [13]. The flow régimes found in the annulus exemplify this generic result, which has wide implications in 'geophysical and astrophysical fluid dynamics' (GAFD). And in view of the effective need for departures from axial symmetry that is implied by the existence theorems for self-exciting dynamos [5] and by Cowling's non-existence theorem [6], the result indicates one possibly key rôle played by Coriolis forces in the geodynamo process and the MHD of the Earth's core.

We note here, in passing, another phenomenon with wide implications in GAFD. This was observed during a brief study made of thermal convection in a rotating *spherical* (rather than cylindrical) annulus subjected to a horizontal temperature gradient and outlined in my PhD dissertation [13]. The study was intended to shed light on the effects on the pattern of motions in the Earth's liquid outer core that the presence of the underlying solid inner core might produce, thereby influencing details of the observed geomagnetic field. The experiments confirmed what general theoretical arguments predicted, namely that owing to Coriolis forces the most striking feature of the flow would be an extensive cylindrical 'detached shear layer' aligned parallel to the rotation axis and girdling the inner spherical surface, touching it along the equator. At sufficiently high rotation rates non-axisymmetric waves appeared on the detached shear layer.

But of more immediate significance during the course of the main experiments was the new dimension they acquired when the geophysicist and mathematician H. Jeffreys commented casually that some of my flow patterns reminded him of large-scale motions in the Earth's atmosphere. (Before losing interest in dynamical meteorology nearly two decades earlier, Jeffreys had made original contributions to the subject, starting when he was sent to work at the UK Meteorological Office during the First World War). So I started reading meteorological literature, handicapped at first by my inability to find dynamical meteorologists in Cambridge from whom I could obtain advice. The applied mathematicians there included several leading theoretical fluid dynamicists, but they found my experimental results 'mysterious'. They evidently preferred laboratory studies focused on the validation of mathematical analyses, at a time when many of the ideas and mathematical techniques needed for interpreting the essentially non-linear behaviour exemplified by my results had yet to be developed.

However, I enjoyed helpful discussions about the atmosphere with E.T. Eady and other dynamicists in the Department of Meteorology at Imperial College, London. And in so far as subsequent developments along our 'path of discovery' are concerned, it was fortunate that around that time the director of the so-called 'Hydro Lab' of the Department of Meteorology of the University of Chicago, D. Fultz, was on sabbatical leave visiting fluid dynamicists and meteorologists in Europe. The Hydro Lab had been established a few years earlier at the initiative of two leading dynamical meteorologists – C.-G. Rossby of the University of Chicago and V.P. Starr of MIT – for the purpose of designing laboratory experiments that might shed light on the general circulation of the Earth's atmosphere.

Fultz told me about his careful literature search for relevant studies, in which he had uncovered reports of qualitative laboratory observations of flows in spinning fluids made by meteorologists F. Vettin (in 1857 in Berlin) and F.M. Exner (in 1923 in Vienna), whose findings had been confirmed by Fultz and his colleagues at the Chicago Hydro Lab in their so-called 'dishpan' experiments [14] – in which the convection chamber was an ordinary domestic aluminium open (American) dishpan. He was understandably interested in my work on the flow régimes obtained in the controllable, geometrically-simple and well-defined annulus apparatus, especially the regular non-axisymmetric régime. With a view to having my apparatus reproduced and my experiments repeated in his own laboratory, he visited me in Cambridge on several occasions in order to obtain details of my results and experimental techniques and of the design and construction of the rotating annulus.

Over the next few years (after I had left Cambridge and was engaged elsewhere in other work), the Hydro Lab confirmed my results and extended the experiments to somewhat lower rotation speeds than those used in the Cambridge studies [15]. And in his successful efforts to bring the experiments to the attention of other meteorologists, Fultz promoted the use of the term 'vacillation' and introduced nomenclature of his own. Thus, the critical dimensionless parameter Θ (see equation (1)) that I had deduced from my experimental data to be the main determinant of the characteristics of the annulus flows [12, 13] he termed the 'thermal Rossby number'; to my regular and irregular non-axisymmetric régimes of sloping convection he gave the single term 'Rossby régime'; and the axisymmetric régime he termed the 'Hadley régime' – after G. Hadley whose celebrated paper on the cause of the Trade Winds was published as early as 1735 [16].

Opinions still vary concerning the meteorological relevance of the laboratory experiments, but from an early stage Lorenz at MIT was amongst those who saw the importance of attempting to identify the dynamical processes underlying the various flow régimes, especially vacillation,[17] and exploring their implications for the predictability of atmospheric motions. To paraphrase his views as expressed in a monograph on the general circulation of the atmosphere [16]:

> So far as their meteorological significance is concerned the experiments, by indicating the flow patterns that can occur and the conditions favourable to each, have made possible the separation of essential from minor and irrelevant considerations in the theory of the global atmospheric circulation. They show, for instance, that while considerations of water vapour may yet play an essential rôle in the Tropics, it appears to be no more than a modifying influence in temperate latitudes, because the hydrodynamical phenomena found in the atmosphere, including even cyclones, jet streams and fronts, also occur in the laboratory apparatus where there is no analogue of the condensation process. The same remarks apply to topographic features, which were intentionally omitted in the experiments. The so-called 'beta-effect' associated with the sphericity of the spinning Earth – which produces a tendency for the relative vorticity to decrease in northward flow and increase in southward flow because of the variation with latitude of the Coriolis parameter – now appears to play a lesser rôle than had once been assumed. Certainly a numerical weather forecast would fail if the beta-effect were disregarded, but the beta-effect does not seem to be required for the production of typical atmospheric systems. The experiments have emphasised the necessity for truly quantitative considerations of planetary atmospheres. These considerations must, at the very least, be sufficient to place the Earth's atmosphere in one of the free non-axisymmetric régimes of thermal convection discovered in the laboratory work.

5. Theoretical Fluid Dynamics and Atmospheric Predictability

Theoretical work in fluid dynamics is based on the nonlinear four-dimensional (space and time) partial differential equations (PDEs) in terms of which the laws of dynamics and thermodynamics can be expressed mathematically. The equations of electrodynamics are also needed in cases of MHD flows in electrically-conducting fluids. Being highly intractable, the equations yield to traditional analytical methods only in simple special cases when nonlinear terms can be neglected or treated as small perturbations.

Recent years have witnessed impressive progress in the application of numerical methods of solution that exploit the power of modern supercomputers, with dynamical meteorologists in centres for weather and climate forecasting amongst those at the forefront of these developments.[18] But much more remains to be done before entirely trustworthy results become obtainable in this way.

The idea of calculating how the weather will evolve, by solving the equations of hydrodynamics using the meteorological data describing the present weather as the initial conditions, goes back to the work by V. Bjerknes and L.F. Richardson in the early twentieth century. A note of caution was issued at the time by H. Poincaré (whose mathematical work on the 'three-body problem' in planetary dynamics had introduced ideas and methods which are now used widely in chaos theory) when he wrote [19]:

> Why have meteorologists such difficulty in predicting the weather with any certainty? Why is it that showers and even storms seem to come by chance, so that many people think it quite natural to pray for rain or fine weather, though they would consider it ridiculous to ask for an eclipse (of the Sun or Moon) by prayer. We see that great disturbances are generally produced in regions where the atmosphere is in unstable equilibrium. The meteorologists see very well that the equilibrium is unstable, that a cyclone will be formed somewhere, but exactly where they are not in a position to say; a tenth of a degree (in temperature) more or less at a given point, and the cyclone will burst here and not there, and extend its ravages over districts it would otherwise have spared. If they had been aware of this tenth of a degree, they could have known of it beforehand, but observations were neither sufficiently comprehensive nor sufficiently precise, and that is why it all seems due to the intervention of chance.

When studying particular aspects of the behaviour of a fluid dynamical system, the governing nonlinear PDEs can be rendered less intractable, albeit less reliable, by simplifying the spatial and/or temporal representation of processes of secondary interest, as in the so-called 'intermediate' theoretical models. And in extreme cases such as the 'low-dimensional' theoretical models (sometimes called 'toy' models) employed when interest focuses on the influence of nonlinearity on temporal behaviour, further simplifications are effected when formulating the model by 'parameterising' all spatial structure. The resulting system is governed by ordinary differential equations (ODEs) needing comparatively modest computers for their analysis, but their solutions can have nothing more than a qualitative bearing on the prototype.

6. Low-Dimensional Models, the Lorenz Equations and Deterministic Chaos

Low-dimensional models bearing on the nonlinear behaviour of self-exciting fluid dynamos are provided by systems of Faraday-disk dynamos. The simplest versions are those introduced in the 1950s by Bullard and T. Rikitake [20]. The autonomous set of nonlinear ODEs in three time-dependent variables that govern the Rikitake system of two coupled disk dynamos was shown in 1962 by D.W. Allan to possess persistent non-periodic (i.e. chaotic) solutions [21]. However, the character of these persistent solutions depends critically on the neglect of mechanical friction in the original Rikitake (and Bullard) systems.

In concurrent research, Lorenz was developing ideas about the use of low-dimensional models in the interpretation of vacillation[17] and other laboratory flow régimes and also about effects of nonlinear processes on atmospheric predictability [2]. His studies of the nonlinear amplification of the effects of tiny errors in meteorological data and its likely consequences for weather forecasting gave rise to the now-familiar term 'butterfly effect', which attracted the attention of writers on popular science as wide interest later developed in the subject of chaos. Using mathematical and computational techniques he investigated a low-dimensional 'toy' model of convection governed by what later became known as the 'Lorenz set' of three (dimensionless) autonomous ODEs, namely:

$$\mathrm{d}x/\mathrm{d}t=a(y-x), \quad \mathrm{d}y/\mathrm{d}t=bx-y-xz, \quad \mathrm{d}z/\mathrm{d}t=xy-cz, \tag{2}$$

which contain two simple nonlinear terms, $-xz$ and $+xy$. Here $x(t)$, $y(t)$ and $z(t)$ are the three time (t)-dependent variables and a, b, and c are positive 'control parameters'. In one of his solution régimes Lorenz found non-periodic behaviour that would be termed 'deterministic chaos' nearly a decade later in mathematical work on nonlinear dynamical systems. Through its impact on the development of ideas in the theory of such systems, the published account of Lorenz's work [2] became one of the most influential scientific papers of the past few decades [22].

The Lorenz equations and other sets of autonomous nonlinear ODEs continue to provide fruitful lines of mathematical research [22]. And in the words of J.D. Barrow [23] writing about the influence of chaos theory on mathematics:

> The mainstream of mathematics has begun to move away from the high ground of extreme formalism to the study of particular problems, notably those involving chaotic nonlinear phenomena, and to seek motivation from the natural world. This is a return to a distinguished tradition for ... there are complementary examples where our study of the physical world has motivated the invention of new mathematics. The contemplation of continuous motion by Newton and Leibniz ... led to the creation of the calculus ... (and) Fourier series arose from the study of heat flow and optics. In the twentieth century, the consideration of impulsive forces led to the invention of 'generalised functions' ... (which) were used most powerfully by Paul Dirac in his formulation of quantum mechanics. ... In recent years this trend towards specific applications has been perpetuated by the creation of a large body of dynamical systems theory, and most notably the concept of a 'strange attractor', as a result of a quest to describe turbulent fluid motions. The growing interest in the description of chaotic change, which is characterised by the very rapid escalation of any error in its exact description as time passes, has led to a completely new philosophy with regard to the mathematical description of phenomena. Instead of seeking more and more mathematical equations to describe a given phenomenon, one searches for those properties which are possessed by almost every possible equation governing change. Such 'generic' properties, as they are called, can therefore be relied upon to manifest themselves in phenomena that do not possess very special properties. It is this class of probable phenomena that are most likely to be found in practice.

7. Nonlinear Stability and Quenching

The disorder and associated lack of predictability of motions in the Earth's atmosphere and also of flows encountered in other nonlinear fluid systems – such as Lorenz's toy model in the chaotic régime [2] and the laboratory annulus in the irregular non-axisymmetric régime [12, 13] – are due to instabilities associated with feedback and coupling. But nonlinear processes can in some circumstances promote *stability* and *order*, rather than instability and disorder.

Such behaviour can be investigated by modifying the feedback and coupling terms in well-known autonomous sets of nonlinear ODEs [24]. Denote by V the 'volume' of that region of $(a,b,c,$etc.$)$ 'parameter space' where instability of equilibrium solutions gives rise to persistent solutions that fluctuate either periodically or non-periodically (i.e. chaotically) and consider the sets obtained by multiplying each of the nonlinear terms in equations[2] by a 'quenching function' q (say). In general $q=q(x,y,z)$, with $q=1$ corresponding to the special case of the Lorenz set. In the representative cases when $q=1-e+ey$ with e ranging from 0 to 1, V decreases monotonically with increasing e and vanishes when $e=1$. Fluctuating persistent solutions are then completely quenched for all values of (a,b,c), leaving only stable steady equilibrium solutions throughout the whole of (positive) (a,b,c) parameter space [24]!

Nonlinear quenching of the chaotic behaviour of the geodynamo associated with modest changes in boundary conditions at the surface of the Earth's liquid core has been invoked to account for the intermittency seen in the irregular time series of geomagnetic polarity reversals over geological time, with intervals between reversals varying from 0.25MY to 50MY [23]. And there are other examples of nonlinear processes promoting stability rather than instability. Such processes underlie the stability of annulus flows in the régime of vacillation, the comparative regularity of large-scale motions in the atmosphere of the planet Mars and the durability of the Great Red Spot and other long-lived eddies in the atmosphere of Jupiter [12, 13, 25].

8. Epilogue

Research environments changed significantly over the four decades since the final stage of our chosen 'path of discovery' was reached, in 1963. Few areas of science have been left untouched by the astonishing growth in power and availability of computers, which now support most research projects including laboratory work on fluid flows and other nonlinear systems. Over the same period, new observations covering many wavelengths in the electromagnetic spectrum, made not only with ground-based instruments but also with instruments mounted on spacecraft, have had a major impact on meteorology, geomagnetism and other geophysical sciences.

Observations of the atmospheres of other planets (Venus, Mars, Jupiter, Saturn, Uranus and Neptune) now influence research in terrestrial meteorology and climatology, just as observations of the magnetic fields of other planets (Mercury, Jupiter, Saturn, Uranus and Neptune) – none of which had been discovered in 1947 at the start of our 'path of discovery' – influence research in geomagnetism. Larmor's prescient views on solar magnetism have been abundantly vindicated by subsequent research [26].

Our 'path' started with the publication of Blackett's theory of the Earth's magnetism, which was testable and timely. Even though the theory turned out to be wrong it led to important new work in other areas of geophysics. His 1947 paper [1] marks the starting point of yet another (better-known) 'path of discovery'. This involved investigations of the magnetism of rocks taken up in the early 1950s by two groups, one at Cambridge led by Runcorn and the other at Imperial College led by Blackett and J.A. Clegg. Using magnetometers of various types – including the highly sensitive astatic magnetometer designed initially by Blackett for testing his theory [10] – both groups investigated fossilised magnetic field directions of igneous and sedimentary rocks collected from several continents. This enterprise provided new evidence in support of ideas concerning continental drift put forward much earlier, in 1915, by A. Wegener, thereby advancing the general acceptance of the ideas by geologists [27] and setting the scene for the emergence towards the end of the 1960s of the remarkably successful theory of plate tectonics.

A brilliant and versatile physicist, Blackett encouraged basic and applied research in all branches of his subject. Many still remember a talk given in 1948 to a student society during which Blackett gave a clear and convincing explanation of the essential physics of magnetohydrodynamic waves, at a time when Alfvén's important new ideas – which in 1970 were recognised by the award of a Nobel Prize – had yet to gain wide acceptance. Those of us lucky enough to hear him lecture at early stages of our careers gained useful insights into the world of physics, and those who would later venture into research were also influenced by his remarks on areas worth avoiding.

The proceedings of this symposium on 'paths of discovery' are expected by the organisers to interest those concerned with the planning of programmes of research. In such exercises it is never easy, of course, to allow for serendipity, making the ideal of moving along 'well-illuminated open paths' rarely achievable in practice. But useful lessons will doubtless be learnt, even though progress towards a discovery often seems like 'moving around in a darkened room and bumping into furniture'.

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