Paths of Discovery
Pontifical Academy of Sciences, Acta 18, Vatican City 2006
www.pas.va/content/dam/accademia/pdf/acta18/acta18-battro.pdf

MICRODISCOVERIES: A FRACTAL STORY. A CASE STUDY OF CREATIVE PATHS AND NETWORKS IN SCIENCE

ANTONIO M. BATTRO

Summary

My thesis is that several cognitive processes engaged in any 'microdiscovery' are common to many scientific tasks even up to major discoveries or breakthroughs. I will analyze the paths of discovery of the 'fractal character of saccadic eye movements' as a case study. A power function: $f=k. A^{-D}$. relates the number of saccades f with their amplitude A. The exponent is interpreted as the fractal dimension D in the sense of Mandelbrot. Experimental measures show that $1<D<2$. In informational terms 1/D could be a measure of the 'temperature of sight'.

I will discuss the importance of 'analogies' in scientific research, the role of 'chance' encounters with relevant information, meetings with people and findings of experimental data. Finally, I will analyze the concepts of innovation, confrontation, modeling, fits and starts, and parsimony in the process of a scientific microdiscovery.

Introduction

Ainsi lecteur, je suis moy-mesmes la matière de mon livre: ce n'est pas raison que tu employes ton loisir en un subjet si frivole et si vain.
(Michel de Montaigne)

This is a psychological study of a scientific discovery of limited relevance and achieved in a short amount of time, but with a long personal story of several decades of research. It deals with my own mental itinerary in the mathematical and experimental pursuit of the fractal nature of a psychophysical law about eye movements, given as an example. Thus the object of my research is a dual object: (1) a mathematical model (the fractal model) and its experimental application on saccadic eye movements, and (2) a psycho-historical account of the creation of that model based upon my documented recollections simultaneous to the scientific pursuit. Benoît Mandelbrot's famous 'How long is the coast of Britain' (1967) proves that you may increase the length of the coast without limit, provided you take smaller and smaller units of measure. The same could happen here. I am afraid my text could grow without limit if I took smaller and smaller details of my journey toward the discovery of a model for saccadic eye movements. In order to avoid this danger, I will restrict my search only to the first levels of analysis. This essay has the purpose of showing the richness of a standard scientific research, the considerable bulk of associations, images, personal encounters, the complexity of the academic network and the many sheets unfolded during the psychogenesis of a scientific idea. My thesis is that several cognitive processes engaged in any 'microdiscovery' are common to every scientific task right up to the major discoveries. But some important points still remain uncertain. Is the work of genius also of the same kind? Perhaps, if we could describe the common ground of several scientific journeys, the peaks of innovation and creativity will be easily measured against some objective level.

Methodology

> *Caminante no hay camino, se hace camino al andar.*
> (Antonio Machado)

I will now describe the methodology I suggest should be followed in a psychogenetic study of discovery.

a) Documentation. I started with the purpose of making short remarks in a log book during the whole process of discovery, and trying to remain objective about places, dates, encounters, ideas, readings, etc. This log has 29 pages in a small (pocket) format, and is handwritten. I carried the notebook with me the whole day during my work. My objective was to become *at the same time* 'the subject and object' of a microdiscovery (a fractal model for saccadic eye movements, in this case) and to enjoy the double *jeu* instead of trying to dismiss one or the other part of it. As a *subject* I was practicing my expertise in eye movements, experimenting, calculating, analyzing data, etc. But, in parallel, I was also the *object* of another experiment, this time in cognition as a *(micro) discoverer* or explorer of some kind, who kept a diary of his journey towards some remote, if any, discovery. In this successful case it took the fractal format of a power function for saccadic eye movements, but many journeys, as we know, even the most simple, may lead to dead ends.

b) Reconstruction. It is essential to have your piece of discovery published in order to 'reconstruct' your own path of discovery, because this is the final text to be confronted as the *terminus ad quem* of your whole endeavor. Once you have finished your scientific task it is time to sit down and retrace your path. You will immediately find that there were several paths and not just one. These paths form a network, but many went loose and some started to diverge. A whole 'landscape of knowledge' will arise under your eyes, even if the final discovery was so tiny as to disappear in the midst of the overwhelming amount of scientific microdiscoveries in your field. But if you retrace your path you will enrich your expertise and your understanding of science in general. And, last but not least, you will offer us a worthy present under the format of a vivid account of an intellectual adventure.

The reconstruction of your path needs some method too. First you might classify the kind or format of the documents as follows:

1. writings, drawings, calculations
2. loose pages or copybooks
3. images and sounds (if videos, records or photographs were taken)
4. different stages of the final text (or different versions, perhaps in different languages)
5. letters to and from the editors, comments of colleagues, referees (including refusals and corrections)
6. conferences, or seminars about your research, before, during and after the publication
7. appearances in media, articles quoting your publication, etc.

Second, classify the 'material contents' of your cognitive journey, and make a catalogue or list of all the items:

1. sources, private and public, remote and recent
2. people, colleagues, friends
3. places, laboratories, libraries
4. trips, fellowships, grants, prizes
5. specific readings and meetings.

Third, write a personal description of the ‘formal contents’, that will include schemata, tables, reflections, etc. about the different cognitive processes (imitation, analogy, deduction, intuition, etc). This is the most difficult part of all the reconstruction, the heart of the matter.

The Sources

Quia minor error in principio magnum est in fine.
(Thomas Aquinas)

My interest in the study of eye movements started as a medical student during my military service in Buenos Aires (1956). I was enrolled as an assistant to the Department of Electro-encephalography of the Hospital Militar Central under the direction of Dr Abraham Mosovich. He taught me how to register ocular movements using common skin electrodes around the eye. The eye works as a dipole (the retina having a negative electric potential of about 1 mV in relation to the cornea) and any ocular movement produces a shift in the electric field of the eye that can be registered by two channels of an EEG. The eye movement angle is a linear function of the potential, for small amplitudes.

After my graduation as a physician (1957) I got a French scholarship (and the next year a fellowship from the University of Buenos Aires) to work with Paul Fraisse at the celebrated Laboratoire de Psychologie Expérimentale et Comparée de la Sorbonne. I worked two full and exciting years with Fraisse in the field of visual perception and I finally presented my thesis for a Doctorat de l’Université de Paris with the title *L’étendue du champ perceptif en fonction du temps d’excitation* (1960). In Paris I used the same technology in electro-oculography that I had learnt in Buenos Aires. I think that the simplicity of the apparatus helped a lot in my research. I published my first paper on eye movements with Fraisse (Battro, Fraisse, 1961) quoted some thirty years later in his book *Des choses et des mots* (1991).

Unexpectedly Jean Piaget made some references to my work in an article related to visual perception in children and adults, published with Vinh Bang in the *Archives de Psychologie* (1961). This was my first academic contact with the great man and it certainly reinforced my enthusiasm for Genetic Epistemology that had already been aroused by the study of *Logique et équilibre* (1957), quoted in my thesis. This book was written by Jean Piaget, Léo Apostel and Benoît Mandelbrot. This was my first contact with Mandelbrot's thinking. A year later I was invited by Piaget to attend his famous weekly seminar at the Centre International d'Epistémologie Génétique in Geneva in 1962. I was then in Switzerland as a student of mathematical logic at the University of Fribourg (I became a fellow of the Centre from 1966 to 1968). In Paris I began to systematically read Piaget who, at that time, published his important volume on perception *Les mécanisms perceptifs* (1961) – too late to be analyzed in my thesis – and back again in Buenos Aires I wrote a *Dictionnaire d'Épistémologie Génetique* (1966, 1972). For my *Dictionnaire* I read the whole work of Piaget and consulted the big corpus of the *Etudes d'Epistémologie Génétique.* The third volume was written by Léo Apostel, Benoît Mandelbrot and Albert Morf with the title: *Logique, langage et théorie de l'information* (1957). In his text 'Linguistique statistique macroscopique' Mandelbrot introduces a 'fraction' 1/B as the index of the 'informational temperature of a text'. It was long before Mandelbrot's impressive breakthrough in Fractal Theory but the idea of measuring the 'temperature' of a collection of discrete events with the help of a fraction became important to me some sixteen years later. I do not remember having studied this fundamental article before, during my stay in Paris, in any case it was not included in the references of my thesis. A long period of latency seems to be a characteristic of many discoveries in science, small or big (Gruber, 1981). And, even more important perhaps is the development in 'fits and starts' of the original idea during an extended period of time.

A Volatile Idea

> *All kinds of reasoning consist in nothing but a comparison, and a discovery of those relations, either constant or inconstant, which two or more objects bear to each other.*
> (David Hume)

In 1972 political and economic reasons forced me to search for work outside my country. Some Brazilian universities accepted my proposal and I began commuting every month by plane from Buenos Aires, where I lived with my wife and three children, to different towns in Brazil. This decade was, in many senses, one of the most productive times of my scientific life. In my monthly air-commuting trips to Brazil during a decade I learned to enjoy working on planes and in airports. I often traveled with a small library of recent books. I was always searching for some new approaches towards visual geometry, a theme I was developing systematically in Brazil with the help of many colleagues. At that time I was engaged in the study and experimental applications of Lie Groups, Riemannian Geometries, Thom Catastrophes, Reggini Visual Perspectives, and the like, in order to understand some new and old perceptual phenomena.

It so happened that on one of those trips we were forced to stay longer at the new Rio de Janeiro airport, on the way back to Buenos Aires. This time I was reading the fascinating book on *Fractals* by Mandelbrot (1975). At the airport I met an old and respected acquaintance, Professor Alberto González Domínguez, one of the most outstanding mathematicians from Argentina. On the plane we did not sit together and at one point I interrupted my friend's dinner to show him the graphics of Perrin's physical Brownian motion and Cauchy's mathematical trails. Mandelbrot wrote about the 'homogenous chaos' of Perrin's graphs. Both highly irregular paths elicited a powerful image in the memory I had of eye movements, in particular the famous A. Yarbus (1967) rendering of the eye scanning the picture of a human face. I compared a Cauchy flight of one of the plates with the saccadic movements I was so used to studying several years before, without hesitation. It turned out that this particular trace was of Fractal Dimension D=1 and it worked as a trigger for the first working analogy between fractals and eye movements. Saccadic movements are also incredibly irregular and their trails amount to a monstrous entanglement during a long visual search. It was natural for me to think of making sense of this essential irregularity. González Domínguez commented: 'Mandelbrot has got a new client!'.

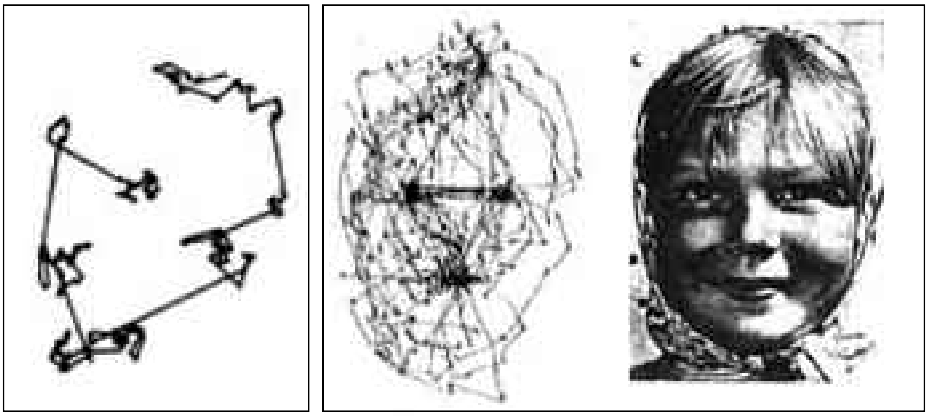


Figure 1. The first analogy: Cauchy flights (left) and eye movements (right) (from Yarbus, 1967).

I must now give some definitions: a *fractal dimension* is a number, not necessarily an integer, that essentially quantifies the degree of irregularity or fragmentation of a geometric set or natural object. A *fractal set* has a fractal dimension that is equal or greater than its ordinary (topological) dimension. A *fractal object* is a natural object that can be represented by a fractal set (Mandelbrot, 1984, p. 154). I rapidly made the hypothesis that the set of saccadic eye movements is a fractal object with a definite fractal dimension D. By analogy with the Cauchy flights on the pictures I postulated a D≥1 for the saccadic movements. And I decided to prove this statement as soon as possible. In retrospect, the first *visual analogy* between fractals and eye movements that triggered the whole process of my microdiscovery was a plate in a book I was reading several thousand feet above the Atlantic Ocean! It was a mental comparison between some *graphics* on paper and a memory of other graphics. Nothing more. The whole story could have ended there, but this particular visual analogy became the beginning and not the end of a fascinating scientific research.

In 1979 I was very pleased to receive an invitation from Fraisse to spend some months in my former Lab as an Associate Director at the Ecole Pratique des Hautes Etudes. I was surprised and honored by this invitation and I decided to explore the 'fractal connection'. As a preview I gave a seminar at the Centro de Investigaciones Filosóficas, CIF, before my trip, where I had the chance to expose my ideas to a group of colleagues. The CIF is a small private research center founded by a group of friends and myself on my return from Europe in 1965. One of the distinguished visitors to the CIF with whom I shared my fractal ideas was the Argentine mathematician Eduardo Ortiz, then director of the Computer Department at the Imperial College in London. Some months later I paid a visit to Ortiz in London to discuss my fractal problems in some depth.

In Paris I began the search for computerized data with experimental findings on frequency (number) of eye movements as a function of their amplitude (in degrees of an angle). Why? Well, my first reading of Mandelbrot's pre-fractal theories (1956) was in the direction of word distribution, Zipf's law, etc., and I was expecting to find a similar (hyperbolic) distribution in eye movements. When we rank the words in a text by decreasing frequency in a sample of one individual's discourse, the result is a near perfect hyperbolic curve of range/frequency. Moreover, for Mandelbrot (1982, p. 345) 'it is sensible to measure how rich a subject's use of vocabulary is through the relative frequency of his use of rare words'. James Joyce's writings, for instance, have a very high 'temperature of discourse', in informational terms.

I hypothesized that the same process may occur during a visual exploration. A 'warm' gaze, a rich visual scanning of an object, should provide a significant increase of large saccades, a fact that I later interpreted as an increase of the 'temperature of sight'. Hence my first, and desperate, quest for rough data about amplitudes and frequencies. I must say that it was not easy for me to find some reliable eye records. Perhaps my question was not relevant enough for my colleagues. I was stubbornly following my arbitrary analogy between words and eye movements but this was a strange and fantastic guess at this stage. Why this 'hyperbolic obsession'? I was under the impression that if not confirmed by some strong experimental evidence then my whole fractal search could abort, and even my stay in Paris could suffer some strain. It is important to note that the dynamic of saccadic movements is not completely under voluntary control, even if we can choose to fix different points in space, saccades are like 'ballistic' movements. In order to understand the magnitude of the amplitudes in a sample, a movement of 5° corresponds to a scanning of 2.5 cm if we explore a picture at 30 cm from the eye. This small movement takes some 30 ms, larger saccades take more time but at greater speed (i.e. 10°: 5 cm, 40 ms, 15°: 7.5 cm, 50 ms).

Imagine a camera taking a very long record of thousands of ocular saccades in a simple journey in the everyday life of an individual. The result will be a terribly messy path of monstrous geometry, covering an enormous amount of points of the visual field! My search was now engaged in the challenge to 'order' this sort of saccadic chaos. The first result is that if we order the saccades by degrees of amplitude, the result is a nice hyperbolic curve! This fact was already known by some experts but I felt that no one paid much attention to the *theoretical consequences* of this very simple phenomenon. S.S. Stevens, the master of modern psychophysics, firmly established 'the surprising simplicity in sensory metrics' (1962). I understood that the same could be said about saccadic metrics.

In the middle of this search for the fractals I received an invitation from Geneva to discuss a quite different subject, my Brazilian findings about brain laterality and Piagetian operations (Battro, 1981). Incidentally, this was my first contribution to the studies on the brain and cognitive development, that became my central objective two decades later (Battro, 2000). In the train I had the intuition of something to be called 'la température du regard'. The day after Piaget's seminar I returned to my obsessive search. I spent many hours at the University Library reading about Zipf's Law and making acquaintance with Vilfredo

Pareto's powerful insights. I read Zipf (1934-1965) and Pareto's very remarkable *Ecrits sur la courbe de la répartition de la richesse* (1896-1965). I was curious to learn from all these hyperbolic distributions and log-log transforms for words and money! They proved to be of great help on my way to understanding the hyperbolic distribution of eye movements too, at least from the point of view of rough data processing. I felt committed to trying a formula myself following Zipf's and Pareto's examples. That evening, October 30th, 1979, exactly at 7.30 p.m., just before going to dinner with Bärbel Inhelder, Piaget's distinguished colleague, for the first time, I wrote the formula that later became:

$f=k.\ A^{-D}$
f: frequency (number of saccades), A: amplitude of saccades
k: a constant, D: a fractal dimension

I was so excited that I wrote a letter to my family in Buenos Aires, trying to explain my finding and of course I joked about the new 'Battro's Law' at dinner.

Next day I had a meeting with two experimental psychologists, A. Bullinger and J.L. Kauffman at the university. We discussed some technicalities about the 'stability' of eye movements. This time I was not speaking about *length* of saccades (amplitude A) but of *directions* in the visual space (angle). At that point I knew that the number of saccades decreased monotonically as a function of their length (amplitude). But what happened with their directions? I was in trouble because I did not understand the mathematics implied in Mandelbrot's discussion on 'stable distributions' and I did not know how to find some consistent data about the distribution of the direction of saccades in visual space. It was pure chance for me that my colleagues had already obtained remarkable computer star-like graphics showing that the eye can reach any point of the visual field in any direction! When every saccade is geometrically translated to a common origin, a quite regular star is obtained, as in Figure 2.

That suggests that – roughly – there are no privileged angles for saccades. Isotropy implies that every angle (direction of sight) has the same probability. This is of course only true between physiological limits, but in the long run, with thousands of saccades, the star will stretch its rays in all the directions of sight. To me this property seemed essential to the fractal model. As a first approximation the star-like results suggest a stable probabilistic distribution for a large sample of saccades. I am now planning to

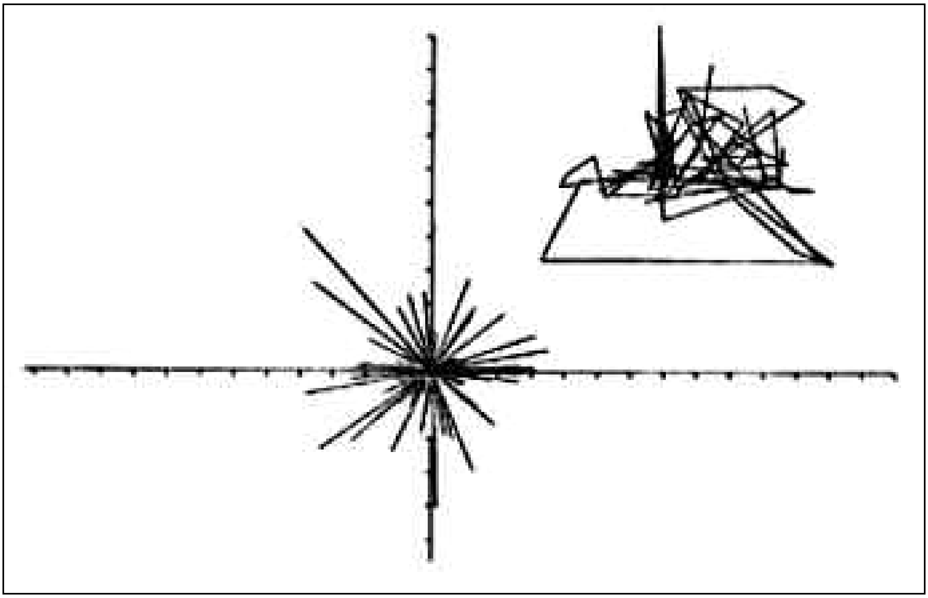


Figure 2. A star of saccades, where eye movements (upper right) are translated to a common origin.

investigate a possible break in this star-like symmetry in some extreme cases of hemianopia (the lack of vision in one half, left or right, of the visual field produced by hemispherectomy; Battro, 2000) more than twenty years after my first observation. A fractal search without end!

That evening, after so much exciting news, I felt very tired. I went to have dinner alone at a fashionable restaurant near my hotel. I dreamt of saccadic stars and power functions. On Thursday night I wrote the first short version of my microdiscovery in French, *La température du regard. Microgenèse d'une idée,* this time at the Café et Restaurant de l'Hôtel de Ville, where I had had so many meetings with friends. Then I went to see 'Le malentendu' the remarkable play by Albert Camus, at the Théâtre de Poche. I wondered if my power function was not also a terrible malentendu. This kind of mixed feelings is very common during the process of discovery, I think. Having reached a harbor, everything has to be put painfully in order again.

Scale and Self-Similarity

> *Chaque portion de la matière peut être conçue comme un jardin plein de plantes et comme un étang plein de poissons. Mais chaque rameau de la plante, chaque membre de l'animal, chaque goutte de ses humeurs est encore un tel jardin ou un tel étang.*
> (Leibniz)

The first step of my research was dedicated to understanding the geometric nature of the saccadic paths, below the superficial image of a messy trail of saccades of different amplitudes and orientations, I was lucky to recognize the deep structure of a simple underlying power function. This was an essential step, of the 'empirical' kind, like Zipf's or Pareto's laws for words or salaries. But I also knew that this finding did not suffice. The better I understood Mandelbrot's theory, the more I became aware of the importance of scaling and self-similarity to complete the fractal model of eye movements, but it was difficult, for me at least, to find a ground for this hypothesis. It is obvious that *scaling* plays a quite different role in mathematics, physics and in eye physiology.

Take the Brownian motion of very fine particles (less than 1 micron) as an example. When its motion is examined under the microscope (see Perrin's *Atoms*, 1909, quoted in Mandelbrot 1977), the successive positions can be marked at very small time intervals and joined by segments. The (constructed) prodigious entangled path left behind is a curve of topological dimension DT=1. Mandelbrot says that when a Brownian motion is examined 'increasingly closely' the trajectory increases without bounds and practically fills the whole plane (dimension D=2)! The disparity between these two values DT<D marks the 'fractal nature' of Brownian motion.

The trajectory of saccadic eye movements can also become a monstrous entanglement but it cannot be compared to a Brownian path because of its physiological nature. The search for scaling structures in nature or society is more difficult than in pure mathematics. Below some lower limit the concept of coastline ceases to belong to geography, (Mandelbrot, 1982) and Vilfredo Pareto (1896-1965) also said that his power law 'non vale per gli angioli'. The same for saccadic movements, I understood that the scaling problem should be tackled from another point of view. In order to find some proof I changed from geometry, my first research objective, to Mandelbrot's *lexicographic trees*. This was a subtle shift indeed, but I was guided by the master's hand.

In fact, Mandelbrot, who made the necessary modifications to the Zipf Law in the fifties, also gave some new insights about D as *a similarity dimension* in the field of linguistics in his 1977 book on fractals. Since the Zipf law of word frequency is near perfectly hyperbolic, I quote from his 1982 version, 'it is eminently sensible to try and relate it to some underlying scaling property. As suggested by the notation, the exponent plays the usual role of dimension. An object that could be scaling does indeed exist in the present case: it is a *lexicographical tree*'. This idea proved enough for me, I tried to represent a saccade of amplitude A as a movement between two fixation points #, for instance #aaa# (saccadic amplitude = 3 degrees). I represented this *saccadic tree* as a simple dichotic branching. In the afternoon I returned again to the laboratory and I discussed this idea with the two experts who provided me with the most fascinating computerized eye movements graphs. They told me that some observations suggested a linear continuum from 'micro-saccades' (of minute amplitude) to 'macro-saccades' of the kind I was examining (larger than 1°). I wrote the first English version (4 pages) of my paper at their Lab. The following day I managed to visit the Nestlé Foundation at Lausanne and a remarkable laboratory of architecture at the University where clients could play with 1-1 maquettes of their future homes. Once again I was immersed in the problem of *scale* that became a central issue of my work as a cognitive psychologist of urban space and open places (Battro & Ellis, 1999).

The next days I continued my analysis of the power function, log-log transforms and the like for a while. Zipf based this most general behavior on the 'principle of the least effort'. I wondered at that time whether this 'principle' could explain saccadic eye movements too. Some months later, I received a charming letter from Mandelbrot telling me that he appreciated my findings, but he was not very sure about Zipf's interpretation of the law of least effort. In the last version of my paper I eliminated it.

A Fractal Flight

Das Wandern ist des Müllers Lust, das Wandern!
(Franz Schubert, Wilhelm Müller)

I returned extremely satisfied to Paris after this week in Geneva and decided to find more experimental data on saccade amplitudes. Imagine my joy, and relief, when my colleague Ariane Lévi-Schoen kindly gave me the experimental answer in the format of computerized hyperbolic distributions of saccades as a function of their amplitude! She provided me with 30 nice computer histograms collected in different experiments on vision by her team. The eye movement samples were automatically collected in small intervals of amplitude. I remember that I was disturbed by a little contradictory data for very small amplitudes in some records, but I was relieved when she told me that these cases were sampling artifacts. These beautiful distributions supported my idea that eye movements also follow (as words do) a hyperbolic function (in this case of frequency/amplitude) at least during the limited amounts of time allowed for eye recording in the laboratory. But I was confident that this was a general phenomenon in all natural settings and for all individuals. I was truly excited! I had found what I was looking for after so many months, the experimental data that could fit into a psychophysical model, a general description for a particular complex motor behavior: the hyperbolic function, so common in so many empirical fields! I started to imagine new interpretations following Mandelbrot's ideas about scaling fractals.

With these findings I took the train *La flèche d'argent,* linked to a short trip on an airplane, to London, to meet my friend the mathematician Eduardo Ortiz at the Imperial College. I wanted to better understand 'the probabilistic machine of hyperbolic curves' that was certainly hidden under the saccadic tree. Ortiz was very kind and helpful. We met at South Kensington. He offered to do the computer calculations for me, if needed, in further eye movement experiments. I was excited when I read in J.R. Pierce *Symbols, signals and noise* (1961) that Mandelbrot observed that as a child grows up the power function exponent, in a sample of his vocabulary, decreases from values of around 1.6 to values of around 1.1. I supposed that a similar developmental study could be done for eye movements. Also I planned to analyze the value of D for different settings and individuals, but unfortunately time ran short and I never had the leisure to do it.

I wrote a first version of my work in 1981 with the title: 'La température du regard. Réflexions sur une démarche scientifique', and sent it to Fernando Vidal, then a young Argentine psychologist graduated from Harvard, now a historian of psychology at the Max Plank Institute in Berlin. The first letter from Vidal reported that Mandelbrot had told him that he had already thought about the similarity between saccades and fractals. This was very encouraging for me, indeed! This reminded me of remarkable synchronies in so many macrodiscoveries. But some were quite disturbing, as in the history of non-euclidean geometries. Gauss boldly wrote to Johann Bolyai's father, that he had already discovered what his son tried to prove: 'to praise it would amount to praise myself; for the results to which he has reached, coincide almost exactly with my own meditations'! In London I decided to send Mandelbrot an English version of my fractal model. I wrote him a letter on the train to Oxford.

On my return to Paris I received an invitation to participate in a Symposium on ocular movements at the Maison des Sciences de l'Homme. Several first class international experts were attending the meeting. I understood immediately that I was not on the same 'longueur d'onde'. At one point I asked to make a comment and I stood up in order to draw the hyperbolic curve of the frequency of saccadic amplitudes on the blackboard and explain my idea of a power function and the fractal model underlying it. It was too premature indeed, and I think nobody took any notice of my proposal. Even Mandelbrot at that time was quite unknown to most psychologists in France.

I also received an unexpected invitation from UNESCO to participate at a meeting in Kathmandu. The meeting was about alternative sciences and epistemologies with the grandiose title of *Meeting of experts on philosophical investigation of the conditions for endogenous development of science and technology* (10-14 December 1979). I went to New Delhi via Constance (where I was invited to give a lecture) and Rome. I realized that with so many flights and stopovers, my trip was a fractal path in itself! As a matter of fact the plane was delayed in Rome until the next day, and we were forced to stay at a hotel near Fiumicino. Instead of making a fuss about this delay I remember how happy I was to have more time to write down my notes and memories. I was really enjoying my explicit decision to make detailed annotations about my personal path towards a microdiscovery. I continued to jot down my memories without interruption during the flight to New Delhi, the day after. I still keep the manuscript of 26 pages I wrote in Spanish. With some delay I met my colleagues in Nepal, where I spent some unforgettable days.

On my return to Paris, Fraisse called me at the hotel to invite me to make a presentation about my findings. I still have the schema of my exposé on an envelope. The envelope also had an interesting story. It contained the first computer graphics I was able to obtain of a family of hyperbolic functions and log-log transformations for decreasing exponents D, a very helpful tool indeed for my empirical work on saccadic distributions. Fraisse suggested that I should publish the core of experimental and mathematical findings without the 'cognitive envelope' of the process of discovery. This suggestion revealed the 'duality' of my work, since I analyzed both the object (the fractal nature of saccades) and the subject (my path of discovery) of my research. I submitted my article in the following months under different versions, short and long, to three top scientific journals in turn, without success. Needless to say I was disappointed by the systematic refusals. But now these failures might take on a new meaning in the context of an essay about the psychogenesis of a scientific idea. I have collected the peer reviews concerning my papers. I must say that all of them helped me a lot to explain some critical points of my work better. This is the great advantage of scientific open criticism and I remain very grateful to these anonymous and dedicated experts. Fortunately, Mandelbrot came to my rescue once again and provided me with invaluable moral support. In 1983 he was confident in my fractal model and wrote: 'I regret that your papers were turned down, but this used to happen consistently to my own works before fractals became the rage'.

A Fractal Revival

> *Beware of notions that are new to be sure, but of which no use can be made after they have been defined.*
> (Lebesgue, quoted by Mandelbrot, 1977)

Thanks to the interest of my colleague Miguelina Guirao, the director of the Laboratorio de Investigaciones Sensoriales LIS in Buenos Aires, in 1993 I dug into my old protocols and notes of 1979 to unearth my fractal model of eye movements. She recalculated some experimental samples and obtained a statistical expression of the negative slope of the log-log transform of the data, with a fractal dimension $D=1.27$ (Figure 3).

As usual, these new and rigorous calculations led me to new insights in the subject matter. I was eager to explore the power of the fractal model and with the help of my young friend Juan P. Garrahan, now a professor of physics at Nottingham University, we produced some simulations of saccadic movements following the two properties, hyperbolic distribution of length of saccades and isotropy of eye movements. Figure 4 is self-explanatory (Battro, 1997).

In June 1994 I returned to Geneva to visit my friends and colleagues, and to attend the Dies Academicus. I returned also to the library in order to search for more data on eye movements. I read the most recent investiga-

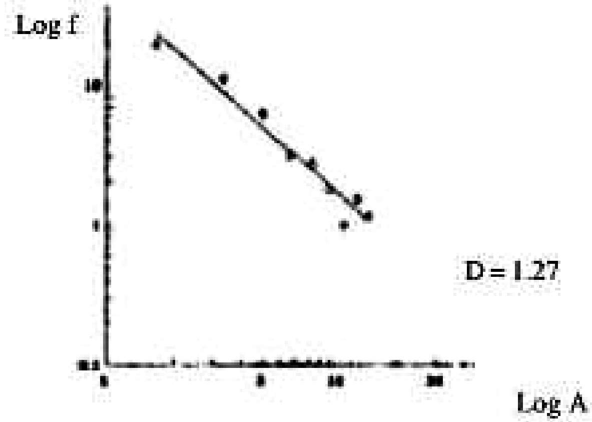


Figure 3. Log-log plot of the experimental data on eye movements. Vertical f: number of saccades, Horizontal: A: amplitude (degrees).

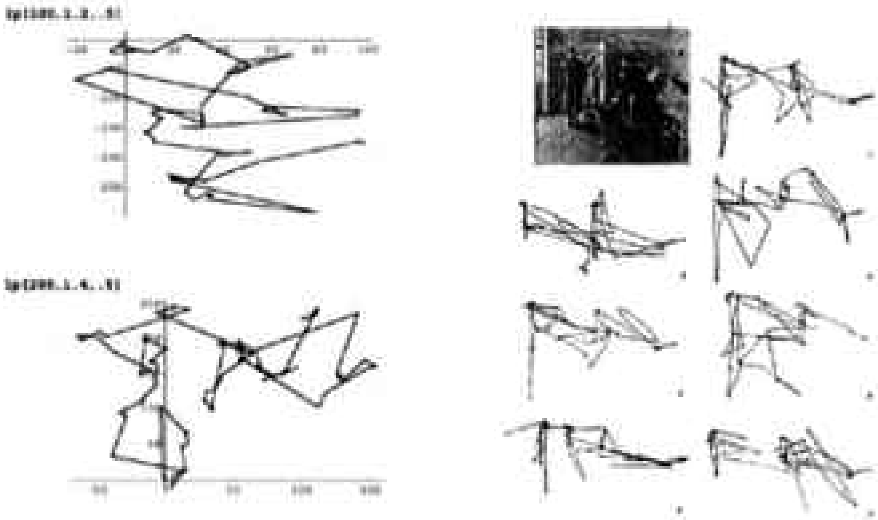
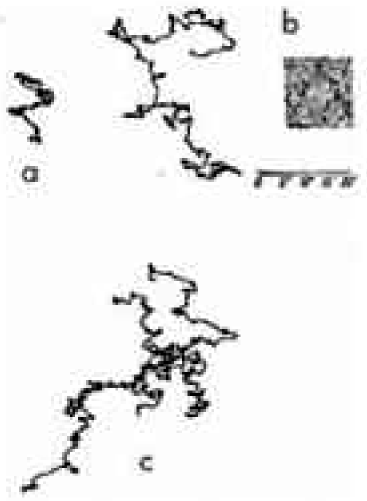


Figure 4. Computer simulation of saccadic eye movements (left). Upper graph: D=1.2, Lower graph: D=1.4. Scanning a picture, from Yarbus (right).

tions without finding any interesting result to add to my old sample on hyperbolic distribution of saccades and very few eye movement records during free search. But I found a most interesting graph in Yarbus' classical book on eye movements. It refers to the microsaccades during fixation on a stationary point. 'The image of the point of fixation always remain inside the fovea… the drift speed *varied chaotically* from zero to approximately 30 minutes of angle per second' (1964). I made a copy of Yarbus' figure 54 and I compared it with Mandelbrot's (1982) figure 255 for a fractional Brown trail of dimensions D~1.11 and D~1.42. As you can see in Figure 5 the traces are remarkably similar in complexity and shape, thus confirming my first vivid impression some decades ago.



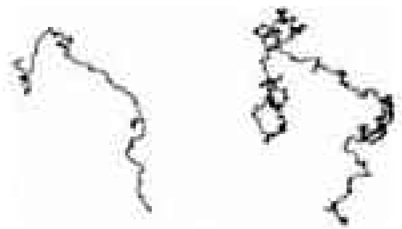


Figure 5. Right: Fractional Brown Trail D~1.11 and D~1.42. Mandelbrot's (1982) Left: three records of eye movements on a stationary point (micro-saccades) a) fixation for 10 sec, b) fixation for 30 sec, c) fixation for 1 min (from Yarbus, 1964).

I discovered two things. First, I returned to my original image, a pure analogy between graphics. This time on a lesser scale, micro-saccades. A new argument perhaps for *fractal space-scaling* (micro-saccades – measured by minutes of angle against macro-saccades – measured by degrees of angle). Second, Yarbus, in retrospect, has given a hint toward the experimental confirmation of *fractal time-scaling* showing three different records for these micro-saccades of 10, 30 and 60 seconds of duration with comparable complexity. Perrin, quoted by Mandelbrot (1982) suggested the property of self-similarity: ‘if the particle positions were marked down 100 times more frequently, each interval would be replaced by a polygon smaller than the whole drawing *but as complicated*, and so on’ (my emphasis). In our case each computer graph of saccades is ‘as complicated’ as the other, for different running times of the simulation.

The Psychogenesis of An Idea

> *From the epistemological point of view we see here how the construction of a new theory is far from being reducible to the accumulation of data, but necessitates an extremely complex structuring of interpretative ideas which are linked to the facts and which enrich them by framing them in context.*
> (Jean Piaget, foreword to H.E. Gruber’s Darwin on Man, 1981)

Now I would like to give some guidelines that might help to follow the genesis of a personal scientific idea:

Agenda: List the people, the places, the letters, the voyages, the drafts, the papers, the conferences, etc. Make a graph with all relevant relations among them. Make a comparison with the story of some important discovery (example: James Watson, *The Double Helix*, 1969). Follow the different paths leading to other microdiscoveries in your scientific life. And try to arrive at some cognitive and epistemological conclusions.

The list of universities where I studied the problem, met relevant people or performed a specific research on fractal eye movements is quite impressive for a modest scientific achievement and it might be compared with some other lists provided by outstanding and mobile scientists during a major discovery. Once again there is a striking commonality between the micro and the macrodiscoveries concerning the number and importance of colleagues and places visited and revisited. I started with a list of 28 people and of the places where I have met them, grouped in 6 classes by the kind of help and expertise (general or specific) they have kindly offered me during my research on eye movements and fractals. It is interesting to add the different personal meeting places related with my research: Buenos Aires, Geneva, Paris, Boston/Cambridge, London. This is good proof of the globalization of science even for a standard research (Suárez-Orozco, Qin-Hillard, 2004). A sociologist of science will also note the predominance of male scientists in my records. I recorded only three women scientists, but their contribution was decisive in many aspects. During this long story five dear masters and colleagues died. Life is broken everywhere at any time, and so are many common and cherished projects. This is a sad fact that must be explicit in every scientific memoir made of flesh and bones. The richer our human network is, the more there are broken threads around us. As for the different documents I conserve on saccadic eye movements and fractals I count 2 manuscripts and 8 papers in different versions (from 1979 to 1994).

And now let us try to analyze 'l'ordre des raisons'. I have found that a substantial part of my research was guided by *analogies*. As a student of philosophy I have read much about the metaphysical importance of analogy. As an epistemologist I was aware of some significant contemporary contributions to the issue, like Mary Hesse's book *Models and analogies in science* (1966). But now, as a scientist, I was confronted by my own insights. My whole research, in fact, was triggered by the analogy between the highly irregular fractal paths discussed by Mandelbrot and saccadic eye movements. Later I developed a kind of mimetic stance with Mandelbrot's formalism, in search of a mathematical expression of the fractal nature of eye movements. I will name the first kind *Image Analogy IA*, and the second *Formal Analogy FA*. I can now summarize the following encounters with analogies on my way to the fractal model:

IA between self-similarity obtained by the change of scale in fractal natural objects (the measurement of the coast of Britain) and in the construction of scale model maquettes of large or small open places.

IA between Mandelbrot's mathematical fractal paths (Perrin, Cauchy flights, etc) and eye movements during a visual search (Yarbus).

FA between some mathematical properties of particular fractal sets, like the hyperbolic distribution in Zipf's and Pareto's laws and the isotropy of space and independence of Cauchy flights.

FA between the fractal dimension D of a text and the fractal dimension D of a sample of saccadic movements.

FA between the informational temperature of a literary text and of a sample of saccades (1/D: the temperature of sight).

FA between Mandelbrot's self-similarity of lexicographic trees and the self-similarity of 'saccadic trees'.

FA between Perrin's self-similarity in time of Brownian movements and the computer simulation of saccadic movements in short and long run.

I must also underline the central role played by *chance* in my whole research. I propose three main categories: Chance encounters with relevant information (CI), chance meetings with people (CP) and chance findings of experimental data (CE). I can order them in a time-line as:

CI Mandelbrot's pre-fractal analysis of Zipf's law in a volume of the *Etudes d'Epistémologie Génétique* (1957) and my link to Piaget and his Center in Geneva (1962).
CI my first reading about self-similarity and Mandelbrot's fractal objects in *Scientific American* (1978). I used this information in my research about the image of the city and the perception of large open spaces.
CI my first contact with Mandelbrot's fractal flights in the pictures of his book on Fractals.
CP my unexpected meeting in Rio de Janeiro with the mathematician González Domínguez on the same plane where I was reading *Les objects fractales* and the chance to discuss the analogy between fractal paths and saccades with him.
CP Fraisse's invitation to spend some months in Paris as Directeur Associé at his Lab and my desire to explore the fractal world of saccades (1979).
CE the starlike graphs of saccades that were registered for other purposes but to me represented the isotropy of the directions of the eye in the visual space (Geneva, 1979).
CE the hyperbolic distribution of saccades given by experimental histograms (Paris, 1979).

I can end this story of a microdiscovery with an overview of some epistemological relevant properties of my quest.

Innovation: the fractal nature of eye movements is a new interpretation of some experimental facts.

Confrontation: the many versions of my scientific paper, the final paper, lectures and seminars, the personal discussion with the experts, the referees' criticisms, show the amount of interaction caused by my model.

Modeling: the hyperbolic distribution of the amplitude of eye movements allows the mathematical calculation of the fractal dimension D. A computer simulation can show different patterns of saccades for different values of D.

Parsimony: The instruments were restricted to standard equipment and simple calculations and programming. Instead, in many major discoveries new conceptual and/or material instruments are constructed. Perhaps this fact is one of the greatest difference among micro and macrodiscoveries.

Fits and starts: The bulk of the theoretical research took place in only three months, although its roots (and branches) expanded over 47 years (from 1957 to 2004) with some critical bursts of creativity, interspersed with long silent periods. I think this finding is interesting and should lead to comparative studies of the different time-lines in scientific and artistic discoveries. It can be related to some basic and cyclical trends of human cognition, that develops in fits and starts too (Fischer and Rose, 1996). In this particular case I can detect the three following spurts of creativity in my search of the fractal nature of eye movements and I can predict a fourth, starting this year.

I: 1958-60, II: 1979-80, III: 1993. IV: 2004

To sum up, my research on visual perception and eye movements of the period 1958-1960 led my quest to finding some reliable data to fit into Mandelbrot's fractal model in 1979. My frantic search, in only a couple of months, of the hyperbolic distribution of the amplitude of saccades and the isotropic distribution of the directions of eye movements, led to the computer simulations of saccades in 1993. In 2002 Fionn Murtagh, professor of computer sciences at the University of Belfast and Mohsen Farid confirmed other aspects of the fractal nature of the eye gaze, following a complete different discovery path and a very sophisticated technique (Murtagh, Farid, 2002). They also applied eye movements to interface with computers, with what they called the 'eye mouse' (Murtagh *et al.* 2003). Finally, they quoted my work. I was happily surprised and grateful.

REFERENCES

Apostel, L., Mandelbrot, B., Morf, A., *Logique, langage et théorie de l'information* (Paris: Presses Universitaires de France, 1957).

Battro, A.M., *L'étendue du champ perceptif en fonction du temps d'excitation*, thèse d'Université (Université de Paris, 1960), 193 pages, unpublished manuscript.

Battro, A.M. & Fraisse, P., 'Y a-t-il une relation entre la capacité d'appréhension visuelle et les mouvements des yeux?', *Année Psychologique*, 61, 313-323 (1961).

Battro, A.M., *Dictionnaire d'Épistémologie Génetique*, preface by J. Piaget (Paris: Presses Universitaires de France, 1966); *Piaget Dictionary of Terms* (Oxford: Blackwell, 1972).

Battro, A.M., 'Hemispheric lateralization in the development of spatial and logical reasoning in left and right handed children', *Archives de Psychologie*, 49, 83-90 (1981).

Battro, A.M., 'A fractal story: the temperature of sight' (1996), http://www.byd.com.ar/fractalstory.htm.

Battro, A.M., 'La temperatura de la mirada', in M. Guirao (ed.) *Procesos sensoriales y cognitivos. Laboratorio de Investigaciones Sensoriales*, Conicet, (Buenos Aires: Dunken, 1997).

Battro, A.M. and Ellis, E.J., *La imagen de la ciudad en los niños* (1999), http://www.byd.com.ar/ciudad.htm.

Battro, A.M., *Half a Brain is Enough: The story of Nico* (Cambridge: Cambridge University Press, 2000).

Fischer, K.W., Rose, S.P, 'Dynamic growth cycles of brain and cognitive development', in R. Thatcher *et al.* (eds.), *Developmental neuroimaging. Mapping the development of brain and behavior* (New York: Academic Press, 1996).

Fraisse, P., *Des choses et des mots* (Paris: Presses Universitaires de France, 1991).

Gruber, H., *Darwin on Man* (Chicago, IL, University of Chicago Press, 1981).

Gardner, M., 'White and brown music, fractal curves and one-over-f fluctuations', *Scientific American*, April (1978).

Hesse, M., *Models and analogies in science* (Indiana: University of Notre Dame Press, 1966).

Mandelbrot, B.B., 'How long is the coast of Britain? Statistical similarities and fractional dimension', *Science* 155, 636-638 (1967).

Mandelbrot, B.B., *Les objets fractals: forme, hasard et dimension* (Paris: Flammarion, 1975).

Mandelbrot, B.B., *Fractals, form, chance and dimension* (San Francisco: Freeman, 1977).

Mandelbrot, B.B., *The fractal geometry of nature* (San Francisco: Freeman, 1982).

Murtagh, F., Farid, M., 'Statistical properties of eye gaze trace data' (2002), http://main.cs.qub.ac.uk/~fmurtagh/eyemouse/distribution/

Farid, M, Murtagh, M., 'Eye-movements and voice as interface modalities to computer systems', in A. Shearer, F.D. Murtagh, J. Mahon, and P.F. Whelan (eds.), *Opto-Ireland 2002*: *Optical Metrology, Imaging, and Machine Vision*, Proceedings of the SPIE, vol. 4877 (2003), pp. 115-125.

Murtagh, F., Taskaya, T., Contreras, P., Mothe, J. and K. Englmeier, 'Interactive visual user interfaces: a survey', *Artificial Intelligence Review*, 19, 263-283 (2003).

Pareto, V., *Oeuvres complètes* (Genève: Droz, 1896-1965).
Piaget, J., Apostel, L., Mandelbrot. B., *Logique et équilibre* (Paris: Presses Universitaires de France, 1957).
Piaget, J.,Vinh Bang, 'Comparaison des mouvements oculaires et des centrations du regard chez l'enfant et chez l'adulte', *Archives de Psychologie* 38, 150, 167-200 (1961).
Piaget, J., *Les mécanismes perceptifs* (Paris: Presses Universitaires de France, 1960).
Pierce, J.R., *Symbols, signals and noise. The nature and process of communication* (New York: Harper-Collins, 1961).
Stevens, S.S., *Psychophysics* (New York, NY: Wiley, 1975).
Suárez-Orozco, M., Qin-Hillard, D.B., *Globalization: Culture and education in the new millennium* (Berkeley: University of California Press, 2004).
Yarbus, A., *Eye movements and vision* (New York: Plenum, 1967).
Watson, J., *The Double Helix* (New York, NY: Norton, 1969).
Zipf, G.K., *The psychobiology of language* (Cambridge, MA: MIT Press, 1965).
Zipf, G.K., *Human behavior and the principle of least effort* (Cambridge, MA: Addison-Wesley, 1949-1965).