PONTIFICIA ACADEMIA SCIENTIARUM

THE AWARD

OF THE

PIUS XI GOLD MEDAL

2004

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The aim of the Pontifical Academy of Sciences, which was founded on 28 October 1936 by the Holy Father Pius XI, is to honour pure science, wherever this may be found, to ensure its freedom, and to support the research essential for the progress of applied science.

On 28 October 1961, on the occasion of the XXVth anniversary of the foundation of the Pontifical Academy of Sciences, the Holy Father John XXIII established the Pius XI Gold Medal in honour of the founder of the Academy. The medal should be awarded to a young scientist who has already gained an international reputation.

The Council of the Academy unanimously decided to award the "Pius XI Gold Medal" for the year 2004 to

Prof. LAURE SAINT-RAYMOND

in recognition of his great merits as a scholar and the important contribution of his research to scientific progress.


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LAURE SAINT
RAYMOND
A.D.
MMIV

LAURE SAINT-RAYMOND

BIOGRAPHICAL DATA

Full Name
Laure SAINT-RAYMOND

Professional Address:
Laboratoire Jacques-Louis Lions
Université Paris VI Pierre et Marie Curie
4 place Jussieu
75013 Paris Cedex 05, France
Telephone: +33 1 44 27 37 72
Fax: +33 1 44 27 72 00
E-mail: saintray@ann.jussieu.fr

Date of Birth: August 4th, 1975

Place of Birth: Paris, France

Citizenship: France

Marital Status:
Married to Sylvain Espinasse
Three children: Clément (10/12/1999),
Thomas (15/07/2001), Lucie (18/06/2003)

Educational Background:

Ecole Normale Supérieure in Mathematics, Paris, France (1994-1998).

Masters in Applied Mathematics, University of Paris VI, France (1996).

Masters in Plasma Physics, University of Versailles, France (1996).

Agrégation in Mathematics (competitive examination for teachers), France (1996).

PhD in Applied Mathematics, University of Paris VII, France (2000).

Accreditation to supervise research, University of Paris VII, France (2002).

Professional Experience:

Research Scientist, Centre National de la Recherche Scientifique (CNRS), Laboratoire d'analyse numérique, Université Paris VI, France (2000-2002).

Full Professor, Laboratoire J.-L. Lions, Université Paris VI, France (2002-present).

Academic Awards and Honors:

Louis Armand Prize, French Academy of Sciences (2003).

Claude-Antoine Peccot Award, Collège de France (2004).

SUMMARY OF SCIENTIFIC ACTIVITY

All along my degree course, I could not bring myself to choose between mathematics and physics, torn between the desire to understand the world that surrounds us and the wish to build some more abstract framework: in the Ecole Normale Supérieure, I could notably enjoy a double training. As a daughter of mathematicians, I have finally opted for the rigour of mathematics but without losing sight of applications. Actually my research is more precisely motivated by plasma physics and meteorology.

My first steps as a researcher, under the supervision of François Golse, were turned onto the study of charged particles submitted to strong constant external magnetic fields (tokamaks, plasmas constituting planetary environments...). From purely mathematical point of view, these works have introduced me to two theories which are still in the heart of my scientific activity: the kinetic theory which allows to model rarefied flows, and the problems of singular perturbations which consist to derive simplified models from a given system in some particular asymptotic regimes. From the point of view of applications, these works have allowed to give a rigorous multiscale analysis of the motion of such plasmas, which is a superposition of fast magnetic oscillations, of an electric transport in the direction of the magnetic field and of a slow drift due to the coupling between magnetic oscillators.

All these results can be easily transposed to the problem of rotating fluids, which is to the study of fluids submitted not to magnetic fields but to the Coriolis force, which is a problem of first order in meteorology. Nevertheless, at some latitudes, the curvature of the Earth has to be taken into account and the approximation of constant penalization for the Coriolis force becomes very false. A series of joint works with Isabelle Gallagher aims at understanding the influence of these homogeneities and in particular to rediscover some trapping phenomena occuring in the equatorial zone.

Among these works, this is probably the one which is the most distant from physicians concerns which has been accorded most attention ... of mathematicians. It is indeed a partial answer, obtained in collaboration with François Golse, to a question asked by Hilbert on the occasion of the International Congress of Mathematicians in 1900. The problem consists actually in getting a unified description of gases from the molecular scale to the macroscopic scale. The mathematical treatment of such a problem can seem if not out of subject, at least unnecessarily complicated, but it allows actually to find in a systematic way some well-known physical phenomena.

From a physical point of view, the problem of fluid limits consists in understanding the links between the various levels of modelisation. The classical models introduced by Euler for inviscid fluids (18th century) and by Navier and

Stokes for viscous fluids (19th century) describe fluids as "continuous" media, thanks to a small number of measurable thermodynamic variables (temperature, pressure, bulk velocity...). The evolution of the fluid is then governed by a system of partial differential equations expressing the local conservations of mass, momentum and energy, system which is closed by a state relation. Such an approach consists then in considering infinitesimal volumes of fluid and to write balance equations for these fictitious particles. This phenomenological approach is actually compatible with the microscopic dynamics of molecules in the fast relaxation limit.

Indeed at the microscopic level the fluid is constituted of a large number of interacting particles governed by Newton's principle. Statistically the collisions between these particles induce a relaxation mechanism: if these collisions are sufficiently frequent, local thermodynamic equilibrium is reached almost instantaneously, which imposes a relation between the various thermodynamic variables, the so-called state relation. And the macroscopic equations of motion are nothing else than the averaged microscopic equations. In the particular case of perfect gases, the volume occupied by the molecules is negligible compared with the volume of the fluid: only binary collisions between uncorrelated particles have a determining role in the evolution of the fluid. In other words a perfect gas can be described by a statistical approach, thanks to its distribution function which gives the instantaneous number of particles of any fixed position and velocity. This distribution function is then governed by a partial differential equation of Boltzmann type.

In the case of perfect gases, the problem of fluid limits asked by Hilbert can therefore be decomposed in two subquestions, the derivation of the Boltzmann equation from Newton's principle (problem solved by Lanford in 1974 for small times) and the derivation of hydrodynamic limits of the Boltzmann equation which is the matter of the works that I would like to present here.

The formal study of hydrodynamic limits of the Boltzmann equation goes back to Hilbert for inviscid perfect gases, and to Chapman and Enskog for slightly viscous perfect gases (the viscosity of perfect gases being necessarily small since the size of particles is negligible). This study lies on the fundamental features of the Boltzmann equation which takes into account both the transport of particles and the statistical effect of instantaneous elastic collisions. The symmetry properties of the collision operator imply actually the local conservations of mass, momentum and energy, and the local increase of some quantity, the so-called entropy. For fixed mass, momentum and energy, the entropy is maximal (and the collision operator cancels) when the velocities of particles are distributed according to a Gaussian, as predicted by Boltzmann.

This means in particular that, if the collisions are sufficiently frequent, the entropy increases rapidly and the distribution of velocities relaxes rapidly to a Gaussian. The state of the gas is therefore completely determined by its thermodynamic fields, which are the temperature, the macroscopic density and the mean velocity. The hydrodynamic equations are then obtained as approximations of some averages of the Boltzmann equation in the fast relaxation limit, that is when the Knudsen number (measuring the ratio between the mean free-path and the typical observation length) is very small.

Depending on the relative sizes of the Knudsen number and of the Mach number (measuring the ratio between the bulk velocity and the thermal velocity), the evolution of the gas is desribed by different hydrodynamic equations. The flow is compressible if the Mach number is of order 1, and incompressible if the Mach number is small. In this last case, the flow is inviscid if the Mach number is large compared with the Knudsen number, and viscous if both parameters are of the same order of magnitude: indeed for perfect gases the Reynolds number (measuring the inverse viscosity) is linked to the Mach and Knudsen numbers through the Von

Karman relation. The mathematical study of each one of these asymptotics is very similar to the study of the corresponding hydrodynamic model: the hydrodynamic approximation leading to the incompressible Navier-Stokes equations is therefore less difficult to understand.

The main diffficulties encoutered to make rigorous the formal derivation sketched previously are actually linked to the physics of the system. The first problem consists in getting a control on particles of high energy: one has to check that such particles are not “lost asymptotically”, or in other words that there is no loss of energy transported by particles which would escape almost instantaneously from the observation domain. In mathematical terms, such a phenomenon would be translated into a loss of compactness (with respect to velocities) on the distribution function, or more precisely on its second moment. The usual a priori estimates based on the entropy and energy bounds do not allow to exclude such a scenario: refined a priori estimates based on the entropy dissipation bound are required to prove that, in the fast relaxation limit, the distribution function has almost the same regularity with respect to velocities as the corresponding Gaussian.

The passage to the limit in the moment equations associated to the Boltzmann equation requires furthermore a precise study of the oscillations created in the system. First of all, one has to check that there is no turbulent effect at the microscopic scale, that is no oscillatory behaviour on lenghts of the order of the mean free-path which would destabilize the whole fluid by some resonance effect. The absence of spatial oscillations is obtained at the mathematical level by some compactness (with respect to positions) on the distribution function: averaging lemmas show indeed that the control on the advection (which is exactly balanced by collisions) gives some regularity (with respect to the space variable) on the thermodynamic fields which are nothing else than averages of the distribution function.

In the same way the system could be destabilized by temporal oscillations (on times of the order of the inverse sound speed, which is supposed to be small). Such oscillations actually take place in the fluid, they are known as acoustic waves (taking into account the weak compressibility of the fluid in such a regime). One has then to check that they do not produce constructive interferences, that is to describe precisely the propagation of the waves and their coupling in the moment equations. An argument of compensated compactness, due to Lions and Masmoudi allows then to conclude that the acoustic waves do not modify the average flow, and consequently that they do not occur in the limiting hydodynamic system.

Besides a mathematical challenge, the rigorous study of hydrodynamic limits is then a way to understand sharp physical phenomena taking place in quasi-hydrodynamic regimes. Therefore it finds out direct applications in the development of technologies requiring precise multiscale numerical simulations (aeronautics for instance). Of course a great number of applications – like medicine – would require a more extended study, considering in particular complex microscopic interactions between elementary particles (blood or breath flows).

LIST OF MAIN PUBLICATIONS

Research articles

Saint-Raymond, Laure. Approximation isentropique du système d'Euler compressible en dimension 1, *C. R. Acad. Sci. Paris Sér. I Math*. **327**, 613–616 (1998).

Golse, François and Saint-Raymond, Laure.. L'approximation centre-guide pour l' equation de Vlasov-Poisson 2D, *C. R. Acad. Sci. Paris Sér. I* Math. **327**, 865–870 (1998).

Golse, François and Saint-Raymond, Laure. The Vlasov-Poisson system with strong magnetic field, *J. Math. Pures Appl.* **78**, 791–817 (1999).

Saint-Raymond, Laure. Hydrodynamic limits for a wave-particle collisions model, *C. R.Acad. Sci. Paris Sér. I Math*. **328**, 837–842 (1999).

Saint-Raymond, Laure. Incompressible hydrodynamic limits for a kinetic model of waves-particles interaction, *Asympt. Anal*. **19**, 149–183 (1999).

Saint-Raymond, Laure. Discrete time Navier-Stokes limit for the BGK Boltzmann equation, *C. R. Acad. Sci. Paris Sér. I Math*. **330**, 163–168 (2000).

Golse, François and Levermore, C. D. and Saint-Raymond, Laure. La méthode de l'entropie relative pour les limites hydrodynamiques de modèles cinétiques, *Séminaire: Equations aux Dérivées Partielles, 1999–2000, Ecole Polytech*. **Exp. No. XIX**, (2000).

Saint-Raymond, Laure. Isentropic approximation of the compressible Euler system in one space dimension, *Arch. Ration. Mech. Anal*. **155**, 171–199 (2000).

Saint-Raymond, Laure. Contrôle des grandes vitesses dans l'approximation gyrocinétique, *C. R. Acad. Sci. Paris Sér. I Math*. **331**, 791–796 (2000).

Saint-Raymond, Laure. The gyrokinetic approximation for the Vlasov-Poisson system, *Math. Models Methods Appl. Sci*. **10,** 1305–1332 (2000).

Golse, François and Saint-Raymond, Laure. The Navier-Stokes limit for the Boltzmann equation, *C. R. Acad. Sci. Paris Sér. I Math.* **333**, 897–902 (2001).

Saint-Raymond, Laure. Dérivation des équations de Navier-Stokes à partir de modèles cinétiques, *Séminaire*: *Equations aux Dérivées Partielles, 2001-2002, Ecole Polytech*. **Exp. No. VI**, (2001).

Saint-Raymond, Laure. Control of large velocities in the two-dimensional gyrokinetic approx-imation, *J. Math. Pures* Appl. (9) **81**, 379–399 (2002).

Saint-Raymond, Laure. Discrete time Navier-Stokes limit for the BGK Boltzmann equation, *Comm. Partial Differential Equations* **27**, 149–184 (2002).

Saint-Raymond, Laure. Du modèle BGK de l'équation de Boltzmann aux équations d'Euler des fluides incompress-ibles, *Bull. Sci. Math*. **126**, 493–506 (2002).

Saint-Raymond, Laure. Un résultat générique d'unicité pour les équations d'évolution, *Bull. Soc. Math. France* **130**, 87–99 (2002).

Golse, François and Saint-Raymond, Laure. Velocity averaging in L^1 for the transport equation, *C. R. Math. Acad. Sci. Paris* **334**, 557–562 (2002).

Saint-Raymond, Laure. Kinetic models for superfluids: a review of mathematical results, *C. R. Math. Acad. Sci. Paris*, *Special Issue*, (2003).

Saint-Raymond, Laure. Convergence of solutions to the Boltzmann equation in the incompressible Euler limit, *Arch. Ration. Mech. Anal*. **166**, 47–80 (2003).

Golse, François and Saint-Raymond, Laure. The Vlasov-Poisson system with strong magnetic field in quasineutral regime, *Math. Models Methods Appl. Sci.* **13**, 661–714 (2003).

Gallagher, Isabelle and Saint-Raymond, Laure. Weak convergence results for inhomogeneous rotating fluid equations, *C. R. Math. Acad. Sci. Paris* **336**, 401–406 (2003).

Gallagher, Isabelle and Saint-Raymond, Laure. Résultats asymptotiques pour des fluides en rotation inhomogène, *Séminaire: Equations aux Dérivées Partielles, 2003-2004, Ecole Polytech*. **Exp. No. III**, (2003).

Saint-Raymond, Laure. From the BGK model to the Navier-Stokes equations, *Ann. Sci. Ecole Norm. Sup. (4)* **36**, 271–317 (2003).

Masmoudi, Nader and Saint-Raymond, Laure. From the Boltzmann equation to the Stokes-Fourier system in a bounded domain, *Comm. Pure Appl. Math*. **56**, 1263–1293 (2003).

Golse, François and Saint-Raymond, Laure. The Navier-Stokes Limit of the Boltzmann Equation for Bounded Collision Kernels, *Invent. Math*. **155**, 81–161 (2004).

Puel, Marjolaine and Saint-Raymond, Laure. Quasineutral limit for the relativistic Vlasov-Maxwell system, *To appear in Asympt. Anal.*, (2004).

Gallagher, Isabelle and Saint-Raymond, Laure. Weak convergence results for inhomogeneous rotating fluid equations, *Submitted*, (2004).

Gallagher, Isabelle and Saint-Raymond, Laure. On pressureless gases driven by a strong inhomogeneous magnetic field, *To appear in SIAM J. Math. Anal.*, (2004).

Golse, François and Saint-Raymond, Laure. The Navier-Stokes limit of the Boltzmann equation with hard potentials, *In preparation*, (2004).

Mischler, Stéphane and Saint-Raymond, Laure. Fast reaction limit of a coagulation-fragmentation equation: strong convergence results, *In preparation*, (2004).

Gallagher, Isabelle and Saint-Raymond, Laure. Strong convergence results for incompressible inhomogeneous rotating fluid equations, *In preparation,* (2004).

Masmoudi, Nader and Saint-Raymond, Laure. The Euler limit of the relativistic Boltzmann equation: convergence towards smooth solutions, *In preparation,* (2004)

Reviews and book chapters

Saint-Raymond, Laure. .Etude mathématique de comportements asymptotiques en dynamique des gaz et des plasmas, *Thèse de doctorat, Université Paris VII*, (2000).

Saint-Raymond, Laure. Méthodes mathématiques pour l'analyse asymptotique de problèmes issus de la mécanique, *Habilitation à diriger des recherches, Université Paris VII,* (2002).

Saint-Raymond, Laure. .Quelques modèles mathématiques en mécanique des fluides, *Cours de DEA, Université Paris VI*, (2003).

Saint-Raymond, Laure. Limites hydrodynamiques incompressibles pour l'équation de Boltzmann, *Cours Peccot, Collège de France*, (2004).

Golse, François and Saint-Raymond, Laure. Hydrodynamic limits for the Boltzmann equation, *Rivista di Matematica della di Parma, Special Issue*, (2004).

Popular scientific works

Saint-Raymond, Laure. De la dynamique moléculaire aux modèles hydrodynamiques: résultats récents sur le sixième problème de Hilbert, *Pour La Science*, (2004).

Saint-Raymond, Laure. Limites hydrodynamiques incompressibles de l'équation de Boltzmann, *Matapli* **73**, (2004).