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RECENT DEVELOPMENTS AND CULTURAL ASPECTS

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For the purpose of this presentation, I will roughly divide mathematics into four large areas: fundamental mathematics, mathematics as the language of theoretical physics, mathematical modeling, and mathematics of modern information and communication technology (computers and networks). This subdivision reflects differences in values more than in content, but exactly for this reason it is relevant for our discussion: science for man and man for science.

1

Fundamental (or pure) mathematics deals with elaborate and subtly interwoven abstractions some of which date back to the ancient Greeks and of which some keep appearing in new issues of mathematical journals. As an example, consider Euler's famous formula $e^{\pi\iota} = -1$. In an extremely concise fashion, it relates three mathematical constants whose respective discoveries were separated by centuries and motivated by very different lines of thought.

In fact, π = 3, 1415926 is essentially a physical constant distilled to a mathematical notion in Euclidean geometry, which itself can be viewed as a physical theory: kinematics of solid bodies in a gravitational vacuum. Unlike π, the base of natural logarithms, e = 2, 718281828, first appeared in the great project which later became computer science, whose purpose was to facilitate time consuming numerical multiplication by replacing it with addition. Finally, the imaginary unit, $i = \sqrt{-1}$ kept emerging, almost against the will of its creators, as an inscrutable but vital fragment of the formal system of algebraic notation, without which it would lack a degree of logical completeness. Euler's formula was a small wonder demonstrating the heuristic power of the recently invented calculus.

As in Euler's time, the proof of a difficult theorem, the solution of an old problem, the formulation of a striking conjecture expressing an insight into an unexplored field, are all important aspects of pure mathematics. Practitioners of pure mathematics teach at universities and pursue their research in a handful of world centers established for this purpose. They constitute a small community in each generation, whose public image, eloquently described by Hans Magnus Enzensberger, is that of insularity and detachment ([E]).

2

As the language of theoretical physics, mathematics enjoys an unusual epistemological status. The point is that the initial semantics and syntax of many mathematical notions often have nothing to do with the secondary semantics which they acquire in the context of a physical theory.

Consider an example: complex numbers and their role in quantum mechanics as "probability amplitudes". Euler's formula, mentioned above (or rather its generalization $e^{i\varphi} = \cos \varphi + i \sin \varphi$), in the context of quantum mechanics explains quantum interference and becomes the quintessence of the wave aspect of the famous wave/particle duality. The imaginary unit, and complex numbers in general, cease to be purely logical constructs and acquire a physical incarnation as quantum phases. They become as "real" as real numbers in the sense that their effects can be experimentally observed and measured. And π in Euler's formula this time does not refer to the angular measure in the Euclidean plane, which is after all only an idealized sheet of paper, but to the considerably more abstract complex plane of quantum phases.

In this way, theories developed in pure mathematics and having their intrinsic logic, are periodically reinterpreted as models of basic physical phenomena. In the last third of this century, this happened to topology and algebraic geometry with the advent of quantum strings, membranes and the project of Grand Unification in the framework of the emerging *M*-theory. Such unpredictable applications are often quoted in order to motivate and justify public spending on pure mathematics. In turn, physics has a profound influence on the development of mathematics. I mention only the recent successes of Feynmann's path integration in topology.

The community of physicists has grown enormously in this century, partly because of military applications, but it is not a purely modern phenomenon. It suffices to recall that Archimedes was a military engineer, and probably, if recognized and captured alive, would have had to work for Romans as Werner von Braun did for the Americans two millennia later.

3

I have set apart mathematical modeling from both pure mathematics and theoretical physics mainly because these activities differ in the minds of those who participate in them. This is discussed at some length by D. Mumford ([M]), who writes: “Models are most prominent in applied mathematics where they express the essential point at which the chaos of experiment gets converted into a well defined mathematical problem”. Galileo’s famous definition of mathematics as the language of nature was based on his experience of such conversion rather than on the fancier mathematical models that emerged later and have a more Platonic flavor.

The distinction between the models used in applied sciences and those of fundamental physics is debatable. In fact, such constructs as the Ising model show that there is no clear-cut boundary between the two. I feel nevertheless that there exists a qualitative difference between, say, the principle of superposition of quantum mechanics (which is a fundamental law) and Hooke’s law stating linear dependence of force as a function of displacement (which is only a convenient model).

More to the point, mathematical model-building includes the vast domain of models in Economics: gathering, processing and distilling statistical data into viable theoretical schemes which often enjoy explanatory and predictive force, but clearly have nothing really fundamental about them. As Mary Poovey convincingly argues in [P], this line of development, starting with Luca Pacioli’s double-entry bookkeeping, informed not only the external forms of modern economic life, but to a considerable degree also the perception of reality and even the self-perception of Western society. Poovey coined the term “modern fact” to express this notion, and thoroughly studied it in her book.

Model-building acquired a new dimension and a large new community of customers with the advent of computers. Perhaps, the most important trait of it is now the option of doing a considerable part of theoretical work by running a program (computer experiment), and/or compiling a vast database (like human genome project). Mathematics of materials (e. g. composites) or models of vision may serve as examples of scientific applications of mathematical modeling.

4

The modern industry of computers and communication networks is the technological embodiment of the abstract mathematical development dealing with the microstructure of information and information processing. The history of this development again spans two millennia, from Aristotle's classification of syllogisms to Turing's machine.

Somewhat paradoxically, with the advent of personal computers and Internet this has become the most visible and most widely used mathematical product. Moreover, user-friendly technological solutions allow one to become a customer of such a system without being burdened by knowledge of mathematics, much in the same way as we drive cars without having to understand thermodynamics and the kinetics of internal combustion.

As I have tried to show, mathematics supported many processes which were vital for the development of modern society and which determined its present state. This role of mathematics raises various issues which I will present in the same order in which I have discussed the sociology of the mathematical community.

5

Pure mathematics traditionally was regarded as a part of high culture, on par with, say, philosophy and music. Edna St. Vincent Millay's line, "Euclid alone has looked on Beauty bare" only a century ago was a poetical expression of commonplace wisdom. This view is now challenged by several cultural shifts.

Academia, with its traditional network of independent universities, libraries, publishing houses and its peer review system, is evolving in the direction of becoming a specialized training and research ground, a part of the service industry subject to market economy laws (especially in the USA), or responsible and directly accountable to government agencies. Consumerism generally lowers cultural standards and tends to dilute or even completely exclude from the curriculum courses requiring hard work and not leading to the immediate and materially rewarding career openings. Mathematics especially suffers from these tendencies.

Applications of mathematics to industry and biology are challenged by the New Age sensibilities. Environmentalists blame science and technology for the destructive uses we made of them, thus further diminishing their cultural appeal.

On the opposite end of the spectrum of intellectual life, deconstructionist and postmodern trends of discourse put in doubt the very notion of scientific truth, trying to replace it by highly arbitrary intellectual constructions, based upon Freudian fantasies and ambiguities of natural language. Moreover, the grand culture of European origin or cultivation is diminished in stature by such pejorative connotations as cultural imperialism and Eurocentrism.

H. Bertens in [Be] argues that postmodernism is essentially positive and compatible with left-liberal political trends, whose essence is "self-reflexivity that leads to the unmasking of prejudices and exclusionist strategies". In particular, in his words, "culture at large becomes aware of its hierarchical structure – with white males at the top of every heap – and begins a long march, far from over, on the road to redressing all sorts of historical wrongs".

These noble intentions notwithstanding, implications of such a mindset for science, its values and its goals, are far from being positive. In fact, they lead to a wild politization of any discourse involving science. In extreme cases they involve misrepresentations of its content and meaning (as was demonstrated by Socal and Bricmont in their analysis of some canonical postmodern texts), and even complete negation of the central notion of science, that of objective truth. As a frustrated Galileo asked in 1605, "What has philosophy got to do with measuring anything?" Four centuries after, we still seem to be unsure about the answer.

This school of thought replaces serious thinking by soul-searching and collective psychotherapy thus effectively preventing our community from seeking responsible answers to the problems of modernity.

Computers and the Internet, helping to solve some of these problems and permitting unprecedented freedom of communication, pave the road to changes, the full scope of which we cannot as yet foresee. As an example, one can cite their role in financial markets which arguably have brought us closer to the dangers of global economic instability. Other concerns are related to the whole structure of information storage and processing in modern society, which is becoming increasingly dependent on quickly mutating and obsolescent hardware and software.

Victor Hugo remarked in "Notre Dame de Paris" that books killed cathedrals (meaning that Enlightenment culture replaced medieval Christian culture). It looks now as if computers are killing books.

Let me quote I. Butterworth: "...the American Association of Computing Machinery, the main US learned society in the computing area, with 18 journals totaling 30000 pages per year, plans to stop completely its printed versions by the year 2000. It is common to hear in such societies statements like 'We want to destroy print'".

There are qualitative differences between libraries and electronic archives. On the positive side, new technology provides the speedy and universal storing of, and access to, information, including papers, books and databases; visual and sound materials; flexible electronic linking; and much more. On the negative side, the life span of the supporting hardware and software is alarmingly short: "...the hardware on which digital information is to be recorded and accessed has a typical life of 10 years" and, moreover, "to a first approximation, no one can use a piece of software written 10 years ago. [...] Maintaining an archive therefore requires expensive human involvement to ensure that, by continual re-copying and by updating of protocols, it can still be accessed. [...] Commercial publishers may be willing and even anxious to maintain archives as long as they have commercial values as databases, but would presumably then allow them to die. If research and scholarship are not to become ephemera, active archives will probably have to be maintained by a combination of learned societies and national legal deposit copyright libraries – but the operation will not be cheap" ([Bu]).

Finally, we are becoming more acutely aware of the dangerous effects of the "stupidity amplification" phenomenon, the Y2K problem being only one of its most outrageous instances.

The fate of mathematics depends on the fate of society. Mathematics was traditionally considered to be the finest expression of rationalism. We must demonstrate again and again that being rational also means being moral.

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