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ASTRONOMICAL APPRECIATION OF THE GREGORIAN CALENDAR (*)

(With two figures)

J. DE KORT S. J.

SVMMARIVM. — Gregoriana calendarii restitutio id constanter enitebatur, ut servatis maiorum traditionibus de statuendae paschatis sollemnitatis ratione veros motus solis lunaeque presse adsequeretur. Quam vero id bene effectum sit e recentibus antiquisque lunae et solis observationibus atque ex theoria motus telluris collata CLAVII explanatione patefit.

INTRODUCTION

The change in the calendar effected in 1582 by Pope GREGORY XIII has been called by its originators a restitution of the calendar rather than a correction (1). As a matter of fact, the Gregorian Calendar not only introduced a scientific improvement at a moment when improvement appeared imperative, but at the same time it also preserved the essentials of older conceptions. In this way was erected a monu-

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(1) Compendium..... ad principes Christianos, 1577; GREGORY XIII, Bull «Inter gravissimas», 1582.

These documents are given by Father CHRISTOPHER CLAVIUS S. J., the semi-official spokesman of the calendar reform, in his book: *Romani Calendarii a Gregorio XIII P. M. restituti Explicatio*. Romae apud Aloysium Zannettum, 1603. The book was reprinted in Clavius' *Opera Omnia*. Notwithstanding its considerable length, it is a readable book even now. I shall cite it by the name of its author.

ment of science as well as of human culture and religious piety, indeed a monument of wisdom in the truest sense of the word. In the present paper, however, we confine our attention to the scientific aspects of the calendar, recognizing of course that the broad wisdom of the authors impregnated even their scientific judgement.

FUNDAMENTALS OF GREGORIAN REFORM

Nowadays most people think that the Gregorian change in the calendar consisted in the suppression of ten days in October 1582 and the dropping, from then on, of three leap-days every four hundred years. Such was not, however, the idea of the originators of the calendar reform. The problem to be solved was a much more complicated one. Connected with the solar calendar by means of the Golden Number, there was the lunar calendar, which had gone slow by about four days and which, like the solar calendar, needed a small constant correction for the future. Every correction of the solar calendar, it was feared, would hopelessly upset the lunar calendar, which was reasonably good only for the existing year of 365,25 days. On the other hand, it seemed undesirable to introduce a wholly new and unknown system into an institution as venerable as the calendar.

It was then that the physician Luigi Gigli proposed to reshape the old epacts into a new system of epacts which could be shifted backward and forward as need should arise. The shifts in one direction, called solar corrections, would insert one day each time into the lunar calendar. They were to take place at the beginning of every year in which a leap-day was to be suppressed in the solar calendar. By the lunar corrections the epacts were to be shifted in the other direction 8 times every 2500 years, each time leaving out one day, to provide a correction of the existing lunar calendar. Gigli called his system a *calendarium perpetuum*, not because he believed that his periods of the sun and moon would never in the future be superseded by better values, but because the correction system devised by him was to be a perfectly flexible tool in the hands of future calendar reformers, since the solar and lunar calendar could henceforth be corrected without mutual interference. It is true that the compensation of the change in the solar calendar by the shift in the epacts is not mathematically perfect. Indeed, for every day dropped from the solar calendar not exactly a day but rather a thirthieth part of a calendar month, viz. about 0,984 days, is reinserted into the lunar calendar. The difference of 0,016 days, however, amounts to a twentieth part of a day only after four hundred years and moreover it can be properly taken care of in the lunar corrections themselves. Any risk that the solar corrections would render the lunar calendar intractable is certainly eliminated.

The actual length of the Gregorian year was based on the value 365,24255 days, which had been adopted in the Alphonsine Tables; the length of the month was taken from COPERNICUS' data as used in the Prutenian Tables. These were what the correctors considered as the most reliable observational values, with the exclusion of periodic changes in the lengths of year and month. The corrections to be applied to the Julian Calendar were rounded off to well-manageable values, viz., the correction of 3/400 days per year for the sun was introduced instead of 0,00745 days, and in the case of the moon 8/2500 days per year was substituted for 0,0031840 days. CLAVIUS states that the rounding-off errors in the calendars for sun and moon will amount to one day by the years A.D. 28400 and 8100, respectively, if the observational values still prove accurate, but he entrusts any further adjustment to the judgement of future correctors of the calendar.

CORRECTNESS OF GREGORIAN YEAR

Comparison of the Gregorian solar calendar with modern astronomical observations is usually made by stating the difference between the calendar and the tropical motion of the mean sun. This is perhaps the most obvious standard by which to judge a calendar constructed according to our current ideas. Such a calendar should be equally accurate, if possible, for every season of the year. From the historical point of view, however, we should first realize which astronomical quantities the Gregorian Calendar intended to reproduce. CLAVIUS states explicitly (Chapter VI, section 8 and elsewhere) that the true sun's transit of the vernal point, not the mean sun's, has to be considered as the fiducial point in measuring the length of the year.

One should keep in mind that the Gregorian Calendar was meant to be an ecclesiastical calendar primarily and only secondarily a civil calendar, and so attention was given mostly to a correct representation of the vernal equinox.

It is apparent, then, that the time interval between two transits of the true sun (¹) over the vernal equinox is now longer than the interval between two transits of the mean sun, because the transit actually takes place in that part of the orbit where the true sun moves faster than the mean sun, and since at the same time the sun's perigee advances relative to the equinox. The table below gives a few numerical values corresponding to the slopes in figure 1. The figure is based on the elements of the sun's motion as adopted by CLEMENCE (²), except for his introduction of Newtonian instead of Universal Time. These elements represent modern and old observations of the sun and the moon, supplemented by theoretical relations between the motions in the solar system.

Tropical Year Expressed in Mean Solar Days

	FOR MEAN SUN	FOR TRUE SUN
1 B. C.	365,2424	365,2422
2000 A. D.	365,2422	365,2423
4000 A. D.	365,2419	365,2423

It is seen that, although the period of the sun's mean motion is gradually decreasing, the effect on the vernal-transit period of the true sun is nearly compensated for by the correction arising from the equation of the center, so that the length of the true sun's vernal

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(¹) The term « true sun » is here used for the sun as affected by its mean motion and the equation of the center only.

(²) G. M. CLEMENCE, *On the system of astronomical constants*. Astr. Journ., 53, 169, 1948.

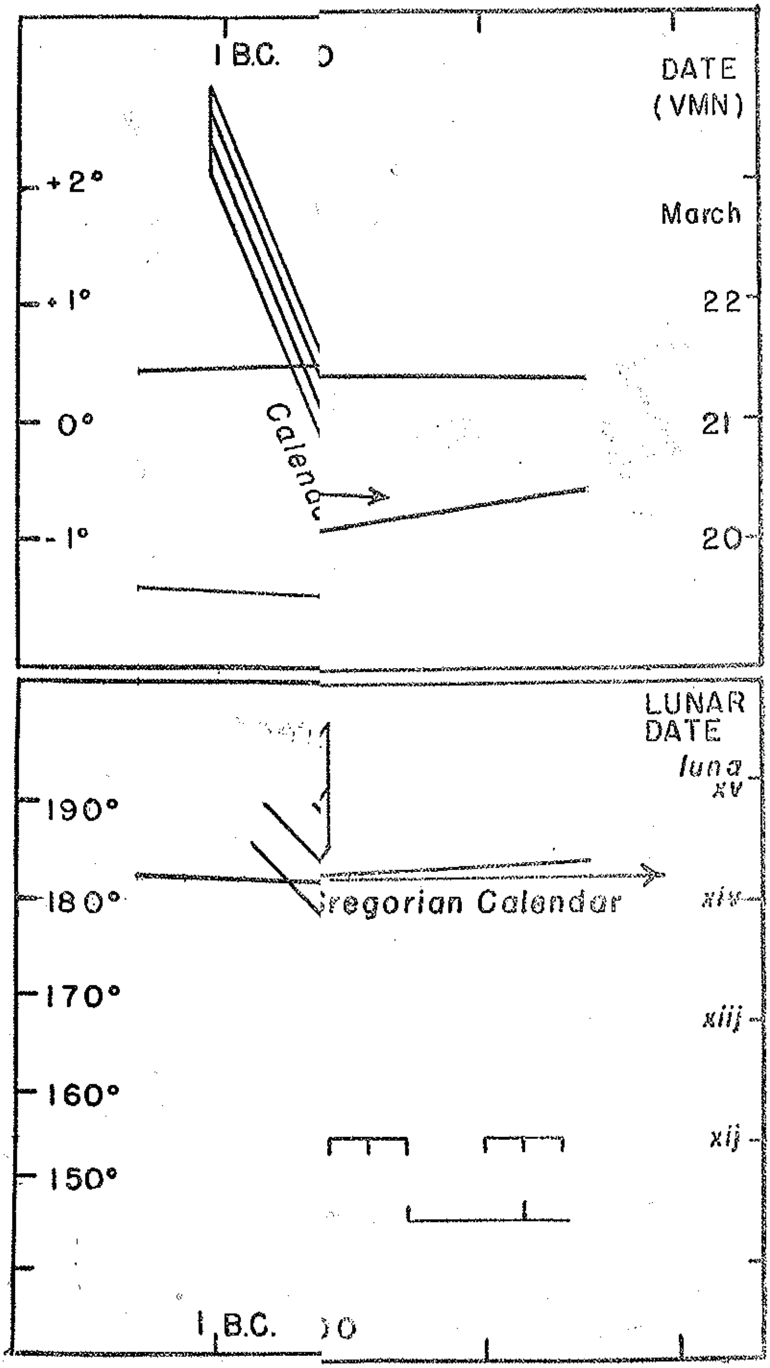
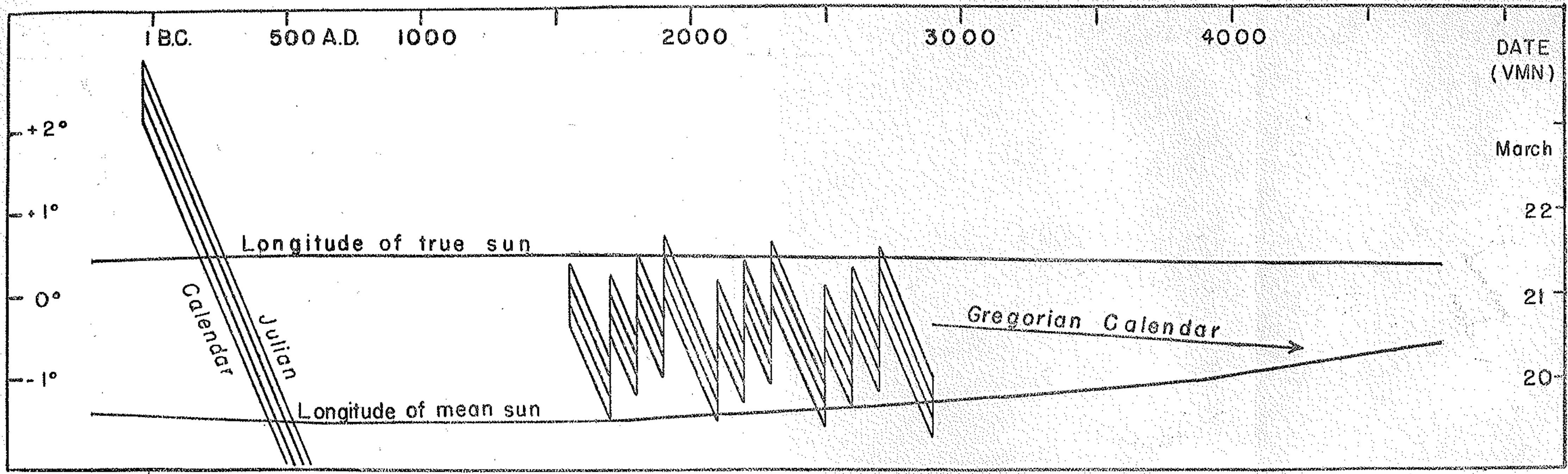


FIGURE 1. Venetian mean noon.



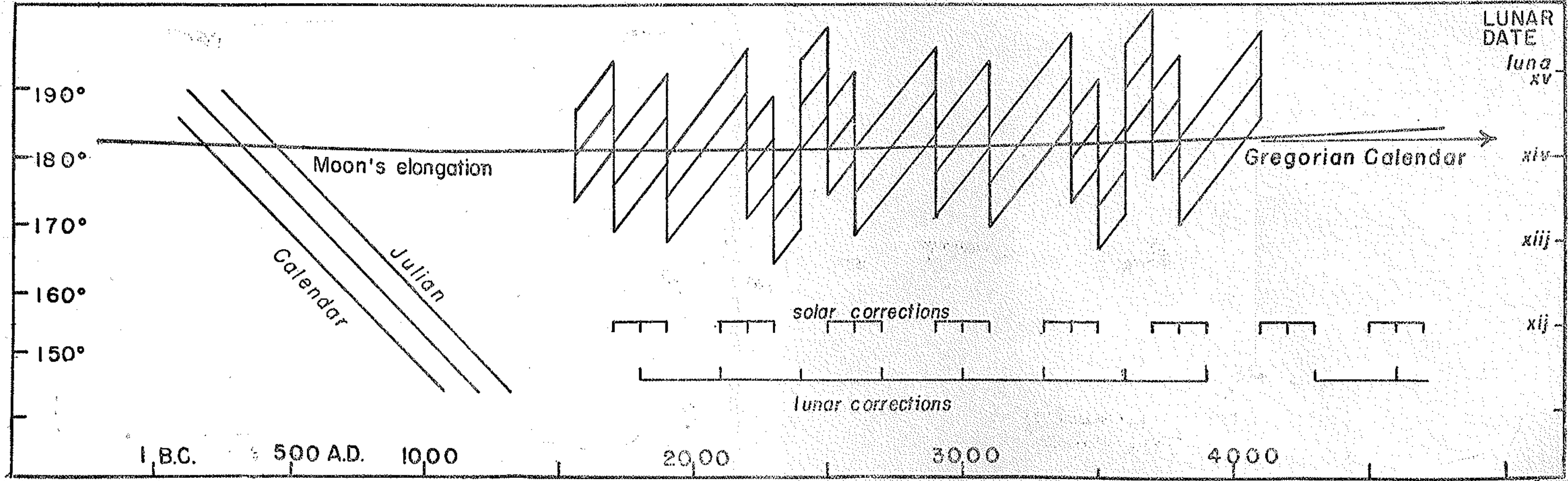


Figure 1. Sun's motion compared with Calendars. — Figure 2. Moon's motion compared with Calendars. — Dates in both figures refer to Venetian mean noon.

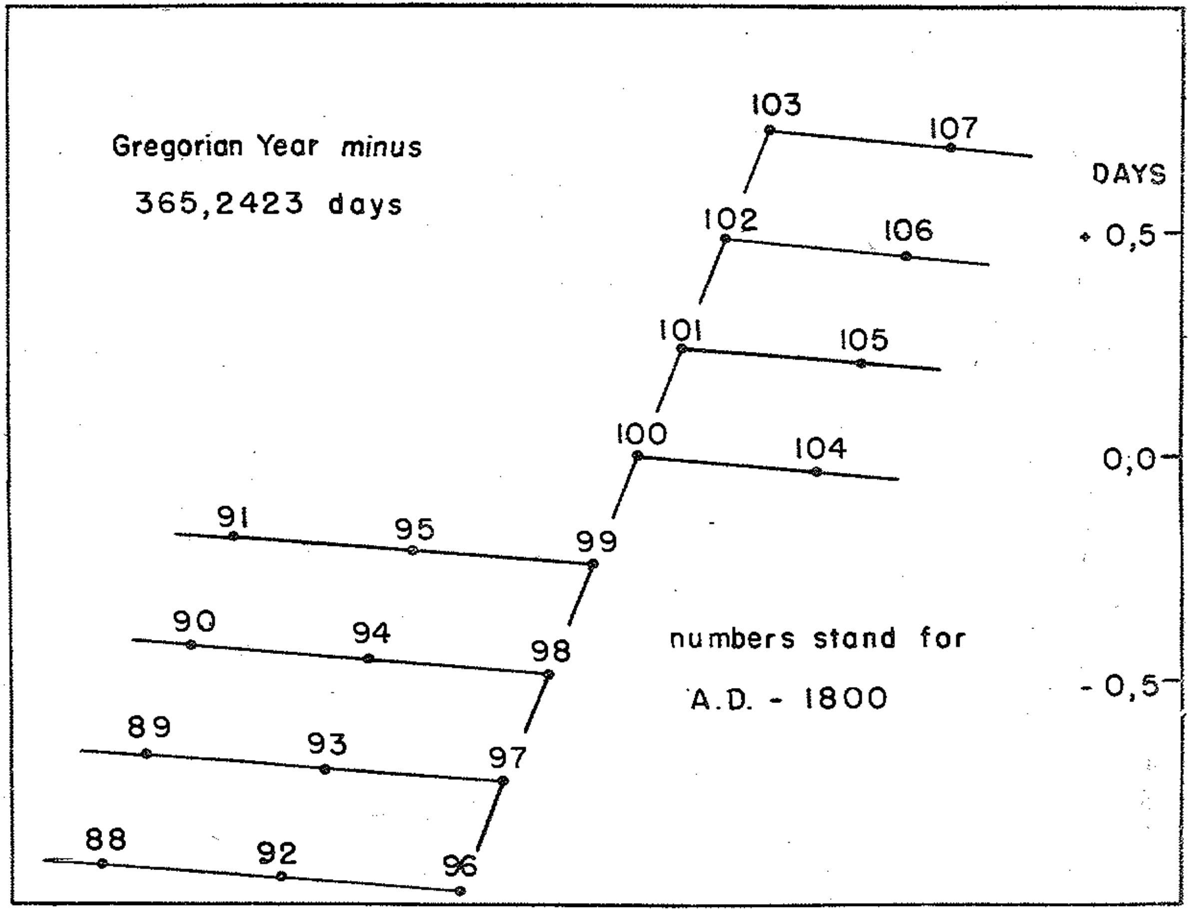


FIGURE 1*a*. – Part of Fig. 1 enlarged.

year is practically unchanged. I am not sure that astronomers in Clavius' time were aware of this difference between the lengths of the two years, though the motion of the perigee was well known to Copernicus (¹). Perhaps the accidental constancy of the true sun's vernal year distracted the astronomers from this difference. The difference in time between the two transits, amounting to nearly two days, is explicitly mentioned by Clavius. He gives much attention to the precessional motion of the equinox which according to Copernicus (²) comprises not only a continuous motion but also a periodic shift with a semi-amplitude of about one degree and a period of 1717 years, resulting in corresponding changes in the length of the year. Clavius does not deny Copernicus' statement, rather he seems to admit its truth; but in his opinion the calendar should not follow these periodic changes but should simply adopt an average length of the year, a decision which would nowadays seem obviously correct. Clavius' reasons are mostly of a cultural or religious character, although he adds that in the present case, since some astronomers still doubt the reality of Copernicus' periodic term, its inclusion would be even less appropriate. The periodicity is inexistent, as we now know, so Clavius' reserve was at the same time sound scientific criticism.

In Figure 1 the longitude of true sun and mean sun are compared with a cycle of artificial years of 365,423 solar days. The cycle is counted backward and forward from 1900 March 21, Venetian mean noon, Venice being the meridian of reference and noon the zero of hour in the Gregorian Calendar. Ordinates for true sun and mean sun are degrees of longitude, and are shown at the left-hand border of the diagram. For the calendars the ordinates are given in days at the right-hand border. In A. D. 2000, when 100 of our artificial years will have elapsed since 1900 March 21, the diagram shows the true sun's longitude and the progress of the calendar as follows. The perpendicular for 2000 cuts the line « Longitude of true sun »

(¹) Nicolaus Copernicus, *De Revolutionibus Orbium Caelestium*. Lib. III, Cap. 20 (numerous editions).

(²) *De Revolutionibus Orbium Caelestium*. Lib. III, Cap. 6, 7.

at + 0,48° and the appropriate line of the Gregorian Calendar figure (the lowest of the four lines, as will be explained below) at March 20, $5^h 31^m$ p. m. We may infer that the transit of the true sun over the vernal point will take place at an instant approximately 0,49 days earlier, i. e. March 20, about $5^h 46^m$ a. m. So in that particular year the calendar will be about 30^h slow, according to the standards of the Gregorian reform. This same interval of 30 hours can be read off directly, if we measure the vertical intercept for 2000 between the true sun's line and the calendar line in terms of the time scale at the right-hand side.

Part of the calendar figure is shown magnified in Figure 1 *a*. The horizontal scale has been slightly more expanded than the vertical. We denote the rise by 0,2423 days after a year of 365 days (from March 21 to March 21) and the descent by 0,7577 days after a year of 366 days. Thus, in the system of four parallel lines the lowest line represents the leap-years such as 1888, 1892, 1896; the second line the following years 1889, 1893 and so on.

The long arrow shows the general trend of the calendar figure. The calendar is apparently going slow, but it will have lost only $8^h 40^m$ in 3582, after 2000 years of the Gregorian Calendar. This comparison is to be contrasted with the usual comparison with the mean sun's year, according to which nearly 20^h are lost in 2000 years, if account be taken of the gradual shortening of the mean sun's year, and $14^h 50^m$, if the present value of the mean sun's year is used.

Figure 1 shows that the calendar figure is nearly always below the line representing the longitude of the true sun, and that the average distance corresponds to roughly 12^h. This implies that the average equinox of the true sun occurs during the night before March 21, Venetian time, as was duly remarked by CLAVIUS. As the earliest possible date for Easter Sunday is March 22, there will be practically no chance of Easter occurring before the equinox.

CORRECTNESS OF GREGORIAN MONTH

Figure 2 graphically represents the moon's synodic motion. The ordinate at the left-hand side gives the difference in longitude, mean moon *minus* true sun. The figure is arranged in essentially the same way as Figure 1, so we need not enter into too much detail. The linear cycle used here is counted backward and forward from 1800 April 9, Venetian mean noon, by multiples of an artificial month of 29,530588 days, where April 9 is the vernal *luna xiv* or calendar full moon for the year 1800. Astronomical data are taken from Schoch's work (¹). The time scale is the same as in Figure 1, the scale in arc being about twelve times as narrow because of the greater angular velocity of the moon. The line system representing the lunar calendar is somewhat simplified. In fact the lunar calendar, apart from the lunar and solar corrections first proposed by Gigli, is subject to continual adjustments of the same character as the quadrennial leap-year adjustments of the sun's calendar, only more complicated. If these minor changes had been individually illustrated in Figure 2, the figure would have shown a great number of different deviations from the average course of the calendar. Instead, groups of three parallel lines are shown. The middle line represents the mean trend of the calendar month as it would be if the Meton cycle were applied to the Gregorian year without any lunar or solar corrections, viz., 19/235 × 365,2425 days. The lowest and highest of the three lines are displaced by a vertical interval of 0,52 days, which is equal to the dispersion of the calendar full moon dates, in spring, about this mean trend. In the left-hand part, showing the Julian lunar calendar, the dispersion is somewhat less, viz., 0,44 days. Towards the lower right-hand corner the centenary years in which lunar or solar corrections take place, have been marked.

According to Clavius, the calendar reformers wished to arrange their corrections to the lunar calendar in such a way that its vernal

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(¹) C. Schoch, *Neubearbeitung der Syzygientafeln von Oppolzer*, Mitt. Astr. Recheninst. Berlin-Dahlem, Bd. 2, Nr. 2, 1928.

luna xiv would not as a rule come much later than mean opposition, and practically never more than a full day earlier than mean opposition. As a result, Easter Sunday, which occurs on the Sunday between *luna xv* and *luna xxj* inclusive, would as a rule not be later than mean last quadrature and practically never earlier than opposition. It will be seen that the arrangement was perfect and will continue to be so for a long time. FOTHERINGHAM (¹) has pointed out that the trend of the Gregorian Calendar is such that it will be faultless at a certain instant near 2200 A. D. In our figure the straight line indicating the Gregorian Calendar trend is exactly parallel to the tangent to the observational curve at the year 2200.

Thus, both the solar and the lunar part of the Gregorian reform constituted at that time a first-rank scientific achievement and the lengths of the year and the month then introduced will presumably meet all ordinary needs of chronology for thousands of years to come, even without the further adjustments which the originators had foreseen.

It is a pleasure to thank Prof. J. TESSER S. J., of Rome, for his interest in the first draft of this note. My thanks are also due to Prof. W. E. VAN WIJK, of Paris, who kindly called my attention not only to certain passages in CLAVIUS and contemporary writers, but also to a diagram of the same type as the one given here, previously published by himself in *De Natuur*.

(¹) J. K. FOTHERINGHAM, the article « Calendar » in the Explanation of the *Nautical Almanac* for the years 1931 to 1938.