POWER SCALING LAWS IN PARTICLE PHYSICS AND ASTROPHYSICS

Rudolf Muradyan

1. Introduction. What is Scaling?

Any power law

$$f(x) = cx^n$$

where exponent n may be a positive or negative number, exhibits the property of *scaling* or scale *invariance*. The word scaling expresses the fact that function $f(x)$ is *shape-invariant* with respect to dilatation (resizing) transformation:

$$x \to \lambda x$$

$$f(\lambda x) \equiv c(\lambda x)^n = \lambda^n f(x)$$

The constant *n* is called degree of homogeneity, and constant c has dimension:

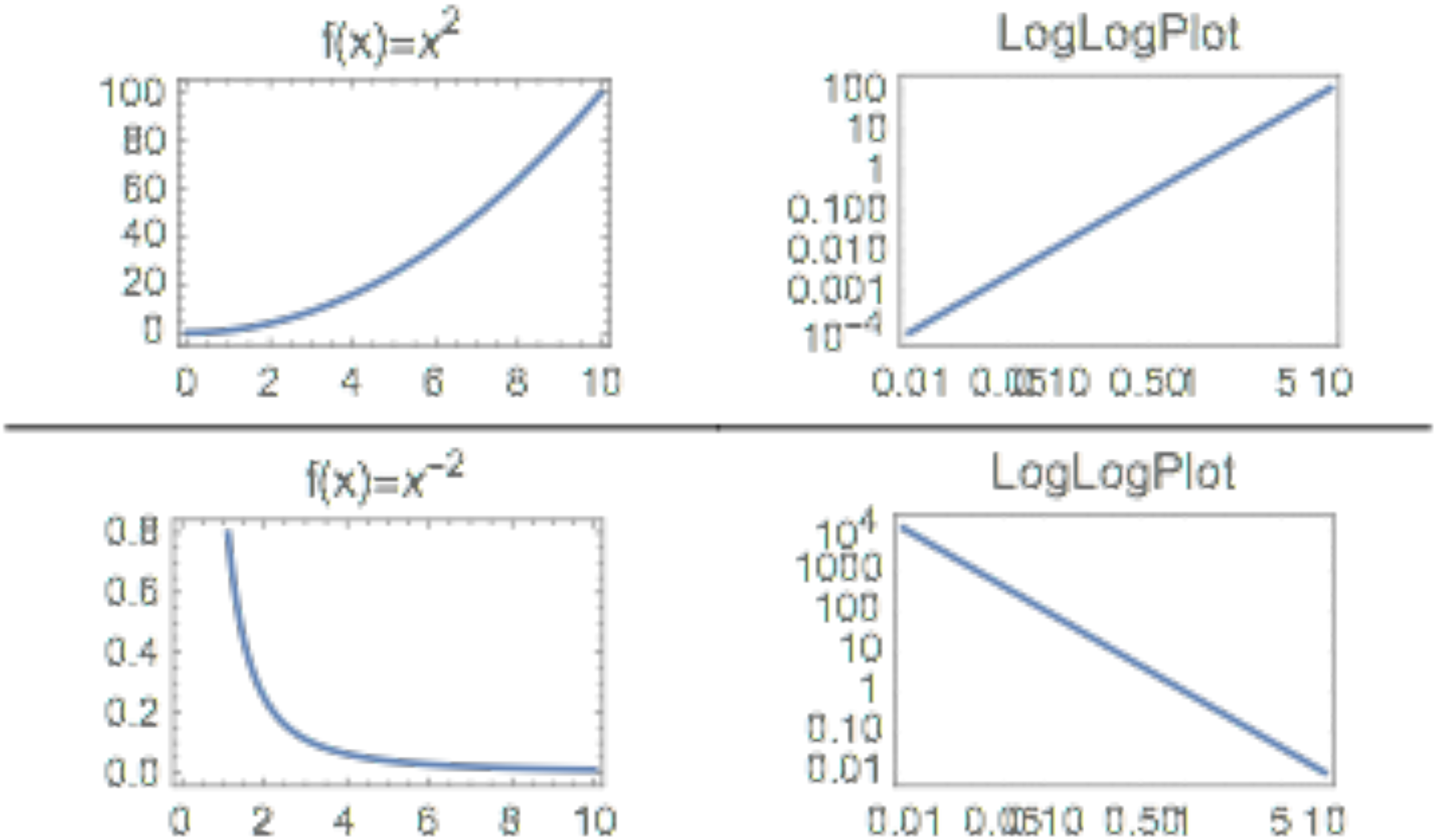
$$\dim c = \frac{\dim f}{(\dim x)^n}$$

Differentiating function $f(\lambda x)$ with respect to λ and putting $\lambda = 1$ we obtain a simple differential equation for scaling function (Euler):

$$x\ f'(x) = n\ f(x)$$

There are a tremendously number of different scaling laws in Nature. A Google search for on the Nobel prize official website www.nobelprize.org picks up nearly 100 results for "scaling". Usually, knowledge of scaling laws is enough to grasp essential characteristics of physical phenomena even without an explicit knowledge of governing equations.

In revealing scaling properties a special role is played by representation of data in double logarithmic or log-log plot. Any scaling curve $y = cx^n$ in x, y plane is possible to recast as a straight line in X, Y plane, where $X = \log x,\ Y = \log y$. Any base can be used for logarithm.
The following is a simple example of scaling in log-log plane:



Now consider a homogeneous function of two variables of degree n:

$$f(\lambda x,\lambda y)=\lambda^n(x,y)$$

By setting $\lambda=\frac{1}{x}$ we obtain an alternative equivalent expression for the homogeneous function of two variables as a product of a power function x^n times some function of one variable:

$$f(x,y)=x^n f\left(1,\frac{y}{x}\right)\equiv x^n\phi\left(\frac{y}{x}\right)$$

Actually all scaling relations are established on this ground. Generalization for many homogeneous variables is obvious.

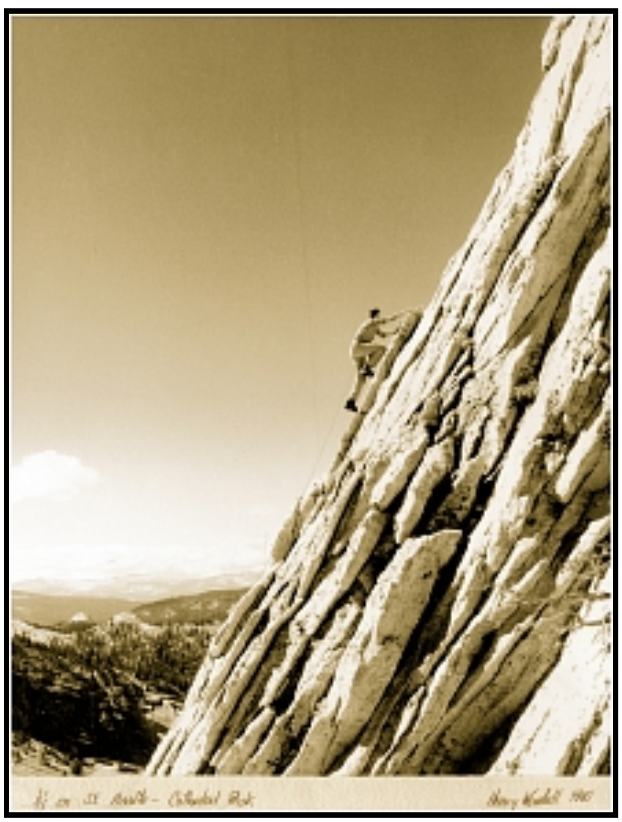
Let us note that any *dimensionless combination* of variables a,b,x,y, say $\left[\frac{a\cdot x^n}{b\cdot y^m}\right]=1$ (or $\pi - group$ according to Buckingham) defines some power scaling law $y=\frac{a}{b}x^\alpha$, where $\alpha=n/m$ [13].

2. Bjorken Scaling

In 1966, prior to SLAC scattering experiments, James «Bj» Bjorken predicted the scaling behaviour for structure functions of deep inelastic scattering (DIS) of electrons on nucleon. This surprising behaviour was found in the data of famous Stanford Linear Accelerator Center (SLAC) experiments and coined as “Bjorken scaling”.

The establishment of Bjorken scaling was one of the most important discoveries in modern high energy physics. It was a direct manifestation of the existence of *quarks* as fundamental constituents of hadrons. Bjorken scaling played a decisive role in the emergence and acceptance of Quantum Chromodynamics (QCD) modern theory of strong hadronic interactions. According to QCD, quarks are permanently bound inside hadrons and probably will never be observed as free particles. Nevertheless, they really exist inside the hadrons. Quarks are fundamental particles in Gell-Mann & Zweig’s *quark model*, proposed in 1964, and constitute the basic blocks of unitary symmetry. Bjorken evaded directly using the name “quark” during his analysis of data. In 1967 he even claimed: “…additional data are necessary and very welcome to destroy the picture of elementary constituents”.

But later experiments persistently provided conformation for quarks existence. Experiments at SLAC performed by H. Kendall, J. Friedman, and R. Taylor confirmed Bjorken’s predictions on scaling and existence of hard point-like constituents inside the proton. Like Rutherford cracked the atom and discovered the proton, J. Bjorken, H. Kendall, J. Friedman, and R. Taylor cracked the proton and unveiled quarks.



James «Bj» Bjorken, SLAC theoretical physicist, at Hawaii Topical Conference, 1985.

James «Bj» Bjorken climbing Cathedral Peak in Yosemite National Park in 1960. (Courtesy of Henry Kendall)

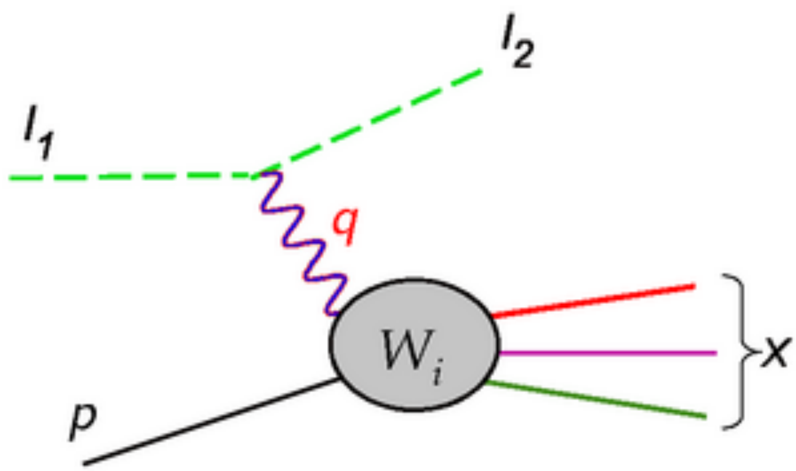
3. Bjorken Scaling and Dimensional Reasoning
(Matveev – Muradyan – Tavkhelidze & T.D. Lee)

The original derivation of Bjorken scaling was performed using current algebra and infinite momentum frame (Bjorken Limit) and was rather complex [1]. Bjorken made a great contribution for clarifying the nature of strong interactions. His radical scaling prediction obtained solid experimental conformation. But there is another approach based on plane dimensional analysis which leads to the same results.

The reaction equation of inelastic scattering of electrons on nucleon can be written as

$$e+p\to e+X$$

where X is the unobserved hadronic system (usually pions), called *missing mass*.
Kinematics of inelastic electron proton scattering is represented below.
If a single electron is detected in the final state, then the cross-section is expressed in terms of two form factors (structure functions) $W_1(\nu,q^2)$ and $W_2(\nu,q^2)$ which depend upon two Lorentz-invariant variables: $\nu=pq=m(E-E')$ is proportional to the energy transfer from electron to hadrons, and $q^2=-4EE'\sin^2\vartheta/2$ is the squared four-momentum transfer or the virtual photon mass.



Let us formulate the scaling (or automodelity) principle. We assume that in describing electromagnetic (or weak) interactions for large energies and momentum transfer none of the dimensional quantities, like masses, "elementary length", etc. are predominant and thus the structure functions depend only upon variable invariants. Therefore, when the scale of measurement of the momentum changes by a factor λ, the structure functions of deep inelastic electromagnetic and weak processes are expected to transform as homogeneous functions of appropriate dimensionality.

It is easy to calculate the dimensionality of the form factors

$$[W_1]=1,\ [W_2]=m^{-2}$$

Under scale transformation $p\to\lambda p,\ q\to\lambda q$ it follows that

$$W_1(\lambda^2q^2,\lambda^2\nu)=W_1(q^2,\nu),\quad W_2(\lambda^2q^2,\lambda^2\nu)=\lambda^{-2}W_2(q^2,\nu)$$

These conditions can be satisfied if we put

$$W_1(\nu,q^2)=F_1\left(\frac{\nu}{q^2}\right)$$

$$W_2(\nu,q^2)=\frac{1}{\nu}F_2\left(\frac{\nu}{q^2}\right)$$

Thus, although W_1 and νW_2 depend, generally speaking, upon two variables, at large q^2 and ν, according to the automodelity or scaling principle, they may become functions of only one dimensionless variable. (In practice it is convenient to use the dimensionless variables ω or x defined according to $\omega=\frac{2\nu}{-q^2}=\frac{1}{x}$. Then in the physical region of electroproduction $1<\omega<\infty$ and $0<x<1$.) Such behaviour of the structure functions of electoproduction was predicted by Bjorken [1] on the basis of the connection of the structure functions W_1 and W_2 with almost equal-time commutators in the limit $q_0 \to i\infty$ and $p_z \to \infty$. This prediction is in rather good agreement with the experimental data. Thus data for different q^2 and ν are described by a single universal curve of nontrivial form.

Let us apply the scaling or automodelity principle to the simplest process $e^+ + e^- \to X$ of annihilation of an electron-positron pair to hadrons. The total cross-section of the reaction is described by a single spectral function $\rho(q^2)$ depending upon single variable q^2 being the square of the energy in the c. m. system:

$$\sigma_{tot}^{ee}(q^2)=\frac{8\pi^2\alpha^2}{q^2}\rho(q^2)$$

Since $\rho(q^2)$ is dimensionless, according to automodelity principle $\rho(\lambda^2 q^2)=\rho(q^2)=C$, where C is a constant. If this constant is not a zero then the annihilation cross section must behave asymptotically as $\sigma_{tot}^{ee}\sim\frac{const}{q^2}$ analogously to the "point" process $e^+ + e^- \to \mu^+ + \mu^-$. In the $x-$ space this prediction leads to a vacuum expectation value of the electromagnetic current commutator being equal to

$$\langle 0\,|[J_\mu(x),J(0)]|\,0\rangle=\frac{iC}{\pi}(g_{\mu\nu}\Box-\partial_\mu\partial_\nu)\delta(\vec{x})\wp\left(\frac{1}{x_0}\right)$$

where $\wp$ is the symbol of the principal value. Hence it follows that the vacuum expectation value of the equal-time commutator between the time and space components $\langle 0\,|[J_0(\vec{x},0),J_i(0)]|\,0\rangle$ is equal to the Schwinger term with quadratically divergent c-number coefficient $\lim_{\tau\to 0}\frac{1}{\tau^2}\frac{iC}{\pi}\nabla_i\delta(\vec{x})$.

It is necessary to note that the dimensional analysis method was independently proposed also by T.D. Lee for derivation of Bjorken scaling in electroproduction and other related processes. In his essay [2] he also refers to our approach. Below is excerpt from T.D. Lee's essay.

T. D. Lee
李政道

Excerpt from T.D. Lee's essay

HIGH ENERGY ELECTROMAGNETIC AND WEAK INTERACTION PROCESSES [2]

Scaling Hypothesis

"The scaling property is the consequence of the scaling hypothesis which was first suggested by Bjorken and others. All the consequences of the scaling hypothesis can then easily be derived by a pure and simple *dimensional analysis*.
See also V.A. Matveev, R.M. Muradyan, and A.N. Tavkhelidze, JINR E2-5962, Dubna, 1971.
I wish to thank J.D. Bjorken for calling my attention to this preprint, in which the authors have also independently emphasized the importance of dimensional analysis in high energy physics".

V. Matveev, R. Muradyan, and A. Tavkhelidze at Bogoliubov Laboratory of Theoretical Physics (1973)

4. Lepton Pair Production in Strong Interactions
(Matveev – Muradyan – Tavkhelidze & Drell-Yan)

The MMT&DY process is a deep inelastic electromagnetic effect when quark and antiquark from colliding hadrons annihilate to create a lepton-antilepton pair [3-6]:

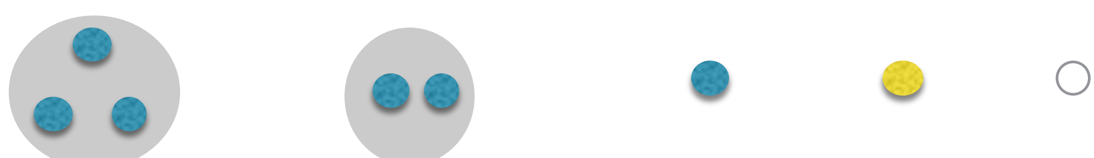
$$a + b \to \mu^+ + \mu^- + X$$

The quarkonium families of resonances J/ψ and Υ were discovered during the study of dilepton spectra in this process.

The tremendous potential of this process in understanding how quarks and gluons are confined inside a hadron, was realized at the new accelerator collider complex NICA,[1] Dubna, Russia. As noted forefather of quark-gluon plasma research T.D. Lee said: "The NICA heavy ion collider will be a very major step towards the formation of a new phase of quark-gluon matter… I am very much looking forward to the completion and future success of the NICA heavy ion collider".

5. Dimensional Quark Counting Rules
(Matveev – Muradyan - Tavkhelidze &Brodsky-Farrar)

According to the quark model, nonexotic barions and mesons consist of three quarks qqq and nonexotic mesons of quark-antiquark pair $q\bar{q}$. Quarks, leptons, and photons are pointlike.



Baryon Meson Quark Lepton Photon

$n_B = 3$ $n_M = 2$ $n_q = 1$ $n_l = 1$ $n_\gamma = 1$

In 1973 V. Matveev, R.M., and A. Tavkhelidze stated that the asymptotic behaviour of inclusive $2 \to 2$ hadronic reactions contains information about distribution and dynamics of *quarks* in hadrons and proposed dimensional Quark Counting Rules (QCR) [7]. Concurrently dimensional QCR was developed by Stanley Brodsky and his collaborators from SLAC.

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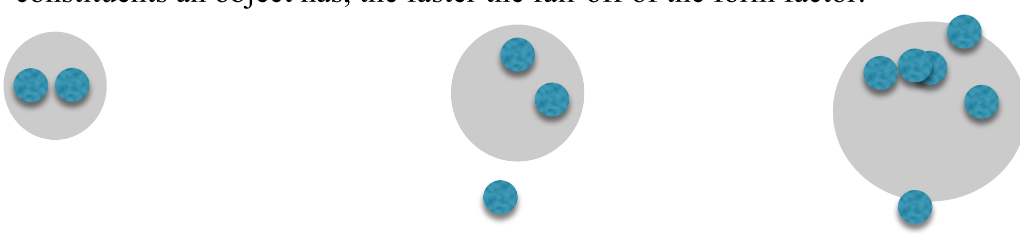
[1] NICA is the abbreviation of Nuclotron-based Ion Collider fAcility.

QCR present a straightforward conformation of validity of the quark model and quark structure of hadrons. Investigations of scaling behaviour in exclusive $2 \to 2$ reactions are similar to the analogous investigation of Bjorken scaling in DIS. The quark model was independently proposed by M. Gell-Mann and G. Zweig in 1964 on the basis of SU(3)-symmetry and generalization of the Sakata model. The Sacata model was a precursor of the quark model and historically the SU(3) symmetry was first introduced in the Sakata model[2] in 1959.

Until the discovery of Bjorken scaling in the SLAC DIS experiments, the total prevailing opinion was that quarks are auxiliary *mathematical devices* for description of SU(3)-symmetry. M. Gell-Mann and G. Zweig, creators of the quark model, were initially proponents of this point of view. QCR played a significant role in clarifying that quarks are real physical entities, not plane mathematical entities. In [7] a simple asymptotic behaviour for electromagnetic form factors was suggested. For composite object H with n_H constituents, the corresponding form factor must behave asymptotically according to QCR:

$$F_H \sim \frac{1}{t^{n_H-1}}$$

where t is the corresponding Mandelstam variable. This relation shows that the more constituents an object has, the faster the fall-off of the form factor.



$F_\pi(t) = \frac{1}{t}$ $\qquad$ $F_N(t) = \frac{1}{t^2}$ $\qquad$ $F_D(t) = \frac{1}{t^5}$

Pion, $n_\pi = 2$ $\qquad$ *Nucleon*, $n_N = 3$ $\qquad$ *Deuteron*, $n_D = 6$

For two-body exclusive reaction $a+b \to c+d$, Quark Counting Rules can be summarized as follows:

$$\frac{d\sigma^{ab\to cd}(s,t)}{dt} = \frac{1}{s^{n_a+n_b+n_c+n_d-2}} f\left(\frac{t}{s}\right) \quad or \quad \frac{d\sigma^{ab\to cd}(s,\vartheta)}{dt} = \frac{1}{s^{n_{tot}-2}} f(\cos\vartheta)$$

where $n_{tot} = n_a + n_b + n_c + n_d$ is the total number of quarks involved in the initial and final states of reaction, s and t are Mandelstam variables, s is square of the total energy in the center-of-mass frame, and t is the momentum transfer squared in the s channel; f is a dimensionless function, depending on the details of the dynamics of the process and $\cos\vartheta = 1 + 2t/s$.

[2] Shoichi Sakata and his Kyoto group (M. Ikeda, S. Ogawa, Y. Ohnuki, Y. Yamaguchi…) were considered *Marxists* and marginalized by mainstream American physicists (as noted by David Kaizer, historian of science).

Some characteristic processes have the following *s* dependence:

$$\frac{d\sigma^{ab\to cd}}{dt} \to \begin{cases} s^{-2} & ee \to ee, \quad \gamma e \to \gamma e \\ s^{-4} & e\pi \to e\pi \\ s^{-6} & ep \to ep, \quad \gamma p \to \gamma p \\ s^{-7} & \gamma p \to \pi p \\ s^{-8} & \pi p \to \pi p \\ s^{-10} & pp \to pp \\ s^{-11} & \gamma D \to np \\ s^{-12} & eD \to eD \end{cases}$$

Thousands of experimental works justify the validity of QCR predictions [8].

6. Huntley's Extension and Scaling in Inclusive Strong Interaction (Matveev – Muradyan – Tavkhelidze)

Huntley in his book (Huntley, H.E., 1967, *Dimensional Analysis*, Dover) pointed out that sometimes it is useful instead of unoriented length dimension L to introduce directed dimensions $L = \{L_x, L_y, L_z\}$.

There are two striking empirical facts about the dynamics of the multihadron production in the collision of two hadrons at high energies:

1. Limited transverse momentum. One of the most surprizing facts of multiparticle hadronic reactions at high energies is the limited range of the transverse momenta, i.e. the magnitude of the component of momentum in the perpendicular plane to the beam direction. The vast majority of the created particles have restricted *transverse* momenta $q_T < 0.4\ Gev.$ While with increasing collision energy *longitudinal* impulses of particles increase $q_z \to \infty$.

On a Peyrou plot of q_T *vs.* q_z with allowed kinematic domain radius $\sqrt{q_T^2 + q_z^2} \le \sqrt{s}/2$ almost all events cluster along the longitudinal z-axis in a strip $q_T < 0.4\ Gev.$

2. Slow particle number growth. The average number of particles $\overline{n}(s)$ grows slowly (logarithmically) with increasing energy. This means that most of the supplied energy is transformed into kinetic energy of longitudinal movement. In other words, interesting physics take place only in the longitudinal direction.

These two facts suggest that there is a strong dynamic difference between the longitudinal and transverse directions. That’s why it is natural to introduce two different scales of length [9-10]:

L_z along the collision axis,

L_T in the transverse plane.

Any physical quantity F, measured in experiments on the collision of hadrons, is characterized by certain dimensions in the longitudinal and transverse directions:

$$\left[F\right] = L_z^n L_T^m$$

Our main scaling hypothesis is the following:

At high energies, there are no fixed parameters having a longitudinal dimension. All basic constants such as the masses, effective radii, and other unknown parameters have purely transverse dimension.

Therefore, under scale transformations of the form

$$q_z \to \lambda q_z, \ \vec{q}_T \to \vec{q}_T$$

any physical quantity should vary as a homogeneous function of the corresponding longitudinal dimension:

$$F \to \lambda^{-n} F.$$

Let us now consider predictions of directed dimensional analysis for specific observables. The simplest inclusive process in two hadron interactions $p_1 + p_2 \to X$ is the measurement of the total cross-section $\sigma_{tot}\left(s\right)$, where $s = \left(p_1 + p_2\right)^2 = 4p_z^2$. By definition, the total cross-section is characterized by a certain effective area perpendicular to the collision axis. Therefore, it is determined by the transverse dimension of length units as follows:

$$\left[\sigma_{tot}(s)\right] = L_T^2$$

The dimension of the invariant s at high energies is purely longitudinal:

$$\left[s\right] = L_z^{-2}$$

Under longitudinal scale transformation according to our main hypothesis we have equality:

$$\sigma_{tot}(s) = \sigma_{tot}(\lambda^2 s)$$

from which it follows that σ_{tot} cannot depend on s, that is

$$\sigma_{tot} = const$$

Of course, logarithmic corrections cannot be captured by this simple method.

Now we consider the predictions of Huntley’s extended dimensional analysis for the *differential* cross section of elastic scattering. At high energies and fixed momentum transfers when $\left[t\right] = \left[\vec{q}_T^{\,2}\right] = L_T^{-2}$ the differential cross-section of elastic scattering has the dimension

$$\left[\frac{d\sigma}{dt}\right] = L_T^4$$

and therefore, cannot depend on s, having a longitudinal dimension

$$\lim_{\substack{s \to \infty \\ t - fix}} \frac{d\sigma(s,t)}{dt} = f\left(t\right).$$

The total elastic cross-section $\sigma_{el}(s)$ and the slope of the diffraction peak $b(s)=\frac{d}{dt}\ln\frac{d\sigma}{dt}\Big|_{t=0}$ are also constant: $\sigma_{el}(s)=const, \quad b(s)=const$.

<u>One particle inclusive distribution</u> $p_1+p_2\to q+X$.

In c.m. system, four dimensional momenta have the following components at high energies

$$p_1=\{p_z,0,0,p_z\}$$
$$p_2=\{p_z,0,0,-p_z\}$$
$$q=\{q_0,\vec{q}_T,q_z\}, \quad q_0=\sqrt{q_T^2+q_z^2}$$

The kinematics of an inclusive reaction $p_1+p_2\to q+X$ can be described by three Lorentz invariant variables

$$s=p_1p_2=2p_z^2$$
$$s_1=p_1q=p_z(q_0-q_z)$$
$$s_2=p_2q=p_z(q_0+q_z)$$

The invariant differential cross-section can be represented as

$$d\sigma=\frac{d\vec{q}}{q_0}f(s,s_1,s_2)$$

Taking into account the dimensions of $[d\sigma]=L_T^2$ and $\left[\frac{d\vec{q}}{q_0}\right]=L_T^{-2}$ we can find the dimension of function $[f]=L_T^4$. Invariant s always has a longitudinal dimension $[s]=L_z^{-2}$ while s_1 and s_2 can have different dimensions in different physical situations. However their product always has a definite dimension $[s_1s_2]=\left[p_z^2(q_0^2-q_z^2)\right]=\left[p_z^2q_T^2\right]=L_z^{-2}L_T^{-2}$.

Now let us consider three different asymptotic regions.

F_1- Fragmentation of 1st particle:

$q_{z\to+\infty}$, $s_1=\frac{p_zq_T^2}{2q_z}$, $s_2=2p_zq_z$ therefore $[s_1]=L_T^{-2}$, $[s_2]=L_z^{-2}$.

F_2- Fragmentation of 2nd particle:

This case follows from F_1 after replacing indexes $1\rightleftarrows 2$, $[s_1]=L_z^{-2}$, $[s_2]=L_T^{-2}$.

$P-$ Pionization region. In this case $|q_z|\sim q_T$ and only the product of s_1 and s_2 has a definite dimension $[s_1s_2]=L_z^{-2}L_T^{-2}$.

In the F_1 region under scale transformation $p_z\to\lambda p_z$, $q_z\to\lambda q_z$, $\vec{q}_T\to\vec{q}_T$ the equality

$$f(s,s_1,s_2)=f(\lambda s,s_1,\lambda s_2)$$

can be fulfilled only if

$$f(s,s_1,s_2)\equiv f\left(\frac{s_2}{s},s_1\right)=f\left(\frac{q_z}{p_z},\vec{q}_T\right)$$

This result also follows from reggeization of $3 \to 3$ elastic amplitude $p_1 + p_2 + \bar{q} \to p_1 + p_2 + \bar{q}$ according to A. Mueller.
In the pionization region from our main scaling hypothesis it follows that:

$$f(s, s_1, s_2) \equiv f\left(\frac{s_1 s_2}{s}\right) = f(q_T)$$

in accordance with double regge expansion of $3 \rightleftarrows 3$ amplitude with trajectory $\alpha_P(0) = 1$.
C.N. Yang and co-workers (*Phys. Rev.* 188, 2159, 1969 and *Phys. Rev. Letters* 25, 1072, 1970) underlined many of these results from the concept of *limiting fragmentation*. R. Feynman (*Phys. Rev. Letters* 23, 1415, 1969) and A. Mueller (*Phys. Rev.* D2, 2963,1970) considered similar problems by different methods.

7. Spin/Mass Scaling for Celestial Bodies

There is geometry in the humming of the strings,
there is music in the spacing of the spheres.
-Pythagoras

Georges Lemaître's Big Bang theory, along with celebrated forecasts, leaves several principal questions unanswered, including the rotation problem.

It is notable that two outstanding members of the Pontifical Academy of Sciences, Monsignor G. Lemaître and, later, Sir Edmund Whittaker considered the *Rotating Primeval Atom* as the possible source of the origin of rotational motions in the Universe.

The central concept of our consideration is that the *Rotating Primeval Atom* must be replaced by the *Rotating Primeval Hadron* with a generalized Regge-like spin/mass relationship. It is amusing that after Chadvick's discovery of the neutron in 1932, Lemaître began to refer to that initial seed as *Neutronic Nucleus*. You can be sure that if he had lived in our times, instead of the rotating A*tom* or N*ucleus* he certainly would have preferred a primeval spinning *hadron*.

Most heavenly bodies, starting from asteroids, planets, and stars to galaxies and clusters of galaxies, possess rotational motion. The rotation of the Sun was observed by Galileo, who attributed the shift of the sunspots to it. Immanuel Kant was the first to suggest that the Milky Way rotates, and this was indeed confirmed by further observations.

In modern astrophysics rotation plays an important role in explaining the emission mechanism of pulsars, which are apparently neutron stars. It has been hypothesized that rapidly rotating dense objects lie at the centers of galaxies and quasars. Finally, indications have recently been found that the entire Universe as a whole may also rotate.

***Monsignor Georges* Lemaître**

"His view is interesting and important not because he is a Catholic priest, not because he is one of the leading mathematical physicist of our time, but because he is both".
- Duncan Aikman, journalist.

Sir Edmund Whittaker

"Rotation is a universal phenomenon; the Earth and all the members of the solar system rotate on their axes, the satellites revolve around the planets, the planets revolve around the Sun, and the Sun itself is a member of the Galaxy or Milky Way system, which revolves in a very remarkable way. How did all these rotatory motions come into being? What secures their permanence or brings about their modification? And what part do they play in the system of the world?
- E. Whittaker

The central point of our consideration is that the *Rotating Primeval* <u>*Atom*</u> must be replaced by the *Rotating Primeval* <u>*Hadron*</u> with a generalized Regge-like spin/mass relationship. The fundamental spin/mass relation *à la* Regge for n-dimensional hadronic objects was proposed in our previous studies [11-13]:

$$J^{(n)}(m) = \hbar \left(\frac{m}{m_p} \right)^{\frac{1}{n}} \qquad n = 1,2,3$$

The number $n = 1,2,3$ characterizes the geometric shape of hadron:

$$n = 1 \qquad \underset{(\text{usual hadrons})}{\textit{string}} \qquad J^{(1)}(m) = \hbar \left(\frac{m}{m_p} \right)^{2}$$

$$n = 2 \qquad \underset{(\text{galaxies})}{\textit{disk}} \qquad J^{(2)}(m) = \hbar \left(\frac{m}{m_p} \right)^{\frac{3}{2}}$$

$$n = 3 \qquad \underset{(\text{stars})}{\textit{ball}} \qquad J^{(3)}(m) = \hbar \left(\frac{m}{m_p} \right)^{\frac{4}{3}}$$

Kerr's spinning black hole is completely characterized by two parameters: its mass m and spin J, connected by relation $J_{Kerr} = Gm^2 / c$. This relation establishes an upper bound on the maximum spin of a black hole.

It is remarkable that Kerr's angular momentum can be obtained from the usual Regge formula for one-dimensional *strings* like hadrons by simple replacement of the proton mass by the Plank mass $m_p \to m_{Pl}$:

$$J_{Kerr}(m) = \lim_{m_p \to m_{Pl}} \hbar \left(\frac{m}{m_p} \right)^2 = \hbar \left(\frac{m}{m_{Pl}} \right)^2 \equiv \frac{Gm^2}{c}$$

where we have used identity:

$$\frac{\hbar}{m_{Pl}^2} \equiv \frac{G}{c}$$

For the reader's convenience let us recall the values of fundamental constants:

$$m_p = 1.673 \times 10^{-27} kg$$
$$\hbar = 1.055 \times 10^{-34} J \cdot s$$
$$c = 3 \times 10^8 m \cdot s^{-1}$$
$$G = 6.674 \times 10^{-11} m^3 \cdot kg^{-1} \cdot s^{-2}$$
$$m_{Pl} = \sqrt{\hbar c / G} = 2.177 \times 10^{-8} kg$$

In the double logarithmic $\log_{10} - \log_{10}$ plot four straight lines represent the following functions:

$$J^{(1)}(m),\ J^{(2)}(m),\ J^{(3)}(m),\ and\ J_{Kerr}(m)\ in\ SI-units,\ with\ [m]=kg\ and\ [J]=J \cdot s$$

$$J^{(1)}(m) = 3.769 \times 10^{19} m^2$$

$$J^{(2)}(m) = 1.542 \times 10^6 m^{3/2}$$

$$J^{(3)}(m) = 53.11\ m^{4/3}$$

$$J_{Kerr}(m) = 2.226 \times 10^{-19} m^2$$

Presently the situation with observational data for galaxies is very controversial. The *pre-dark matter* data are displayed in all $J - m$ plots. The observational data for planets and stars remain unchanged. For details see [11-13].

Kerr's momentum plays an interesting theoretical role in our approach. It helps reveal important relations for limiting mass and spin of cosmic bodies at intersections with Regge trajectories $J^{(2)}(m)$ and $J^{(3)}(m)$. Thereby on the plane (m, J) we discovered two fundamental points with coordinates expressed simply by means of fundamental constants $\hbar, c, G, m_p$. We proposed to name these points as *Eddington and Chandrasekhar points*. Solving equation $\frac{Gm^2}{c} = \hbar \left(\frac{m}{m_p} \right)^{\frac{3}{2}}$ for variable m we obtain an Eddington expression for the mass of Universe, and from the equation

$\frac{Gm^2}{c} = \hbar\left(\frac{m}{m_p}\right)^{\frac{4}{3}}$ the celebrated Chandrasekhar expression for limiting mass of stars follows:

$$m_{Universe} = m_p\left(\frac{\hbar c}{Gm_p^2}\right)^2, \quad m_{star} = m_p\left(\frac{\hbar c}{Gm_p^2}\right)^{\frac{3}{2}}$$

Expressions for Eddington and Chandrasekhar masses via fundamental constants are considered as jewels of theoretical physics and astrophysics. They concern fundamental properties of our Universe. Chandrasekhar's name was immortalized in connection with the formula for m_{star}. In his Nobel lecture he asks "*Why are the stars as they are?*" and responds that it is because their masses are given by combinations of fundamental constants given by the formula for m_{star}.

Expressions for spins of stars and Universe J_{star} and $J_{Universe}$ are relatively *new* and can be obtained from our theoretical Regge formulas for $J^{(2)}(m)$ and $J^{(3)}(m)$ by simple substitutions $m \to m_{Universe}$ and $m \to m_{star}$. Corresponding relations for spins can easily be derived by these substitutions:

$$J_{Universe} = J^{(2)}(m_{Universe}) = \hbar\left(\frac{\hbar c}{Gm_p^2}\right)^3, \quad J_{star} = J^{(3)}(m_{star}) = \hbar\left(\frac{\hbar c}{Gm_p^2}\right)^2.$$

It is convenient here to summarize our main results, which can be represented in three different equivalent forms:

$Eddington\ point\ \{m_{Universe}, J_{Universe}\}$	$Chandrasekhar\ point\ \{m_{star}, J_{star}\}$
$\left\{m_p\left(\frac{\hbar c}{Gm_p^2}\right)^2,\ \hbar\left(\frac{\hbar c}{Gm_p^2}\right)^3\right\}$	$\left\{m_p\left(\frac{\hbar c}{Gm_p^2}\right)^{\frac{3}{2}},\ \hbar\left(\frac{\hbar c}{Gm_p^2}\right)^2\right\}$
$\left\{m_p\left(\frac{m_{Pl}}{m_p}\right)^4,\ \hbar\left(\frac{m_{Pl}}{m_p}\right)^6\right\}$	$\left\{m_p\left(\frac{m_{Pl}}{m_p}\right)^3,\ \hbar\left(\frac{m_{Pl}}{m_p}\right)^4\right\}$
$\left\{m_{Pl}\left(\frac{m_{Pl}}{m_p}\right)^3,\ J_{Pl}\left(\frac{m_{Pl}}{m_p}\right)^6\right\}$	$\left\{m_{Pl}\left(\frac{m_{Pl}}{m_p}\right)^2,\ J_{Pl}\left(\frac{m_{Pl}}{m_p}\right)^4\right\}$

The following identity can been used $\frac{m_{Pl}}{m_p} \equiv \sqrt{\frac{\hbar c}{Gm_p^2}}$ during transformation of these equations. It must also be noted that $J_{Pl} = \hbar$ because of $J_{Pl} = m_{Pl}v_{Pl}r_{Pl} = \sqrt{\frac{\hbar c}{G}}c\sqrt{\frac{\hbar G}{c^3}} = \hbar$.

Let us consider the double logarithmic J, m plane. Any power function $y = cx^n$ in x, y plane can be recast as a straight line in the X, Y plane, where $X = \log x,\ Y = \log y$. In this log-log plot theoretical spin/mass relations represents straight lines.

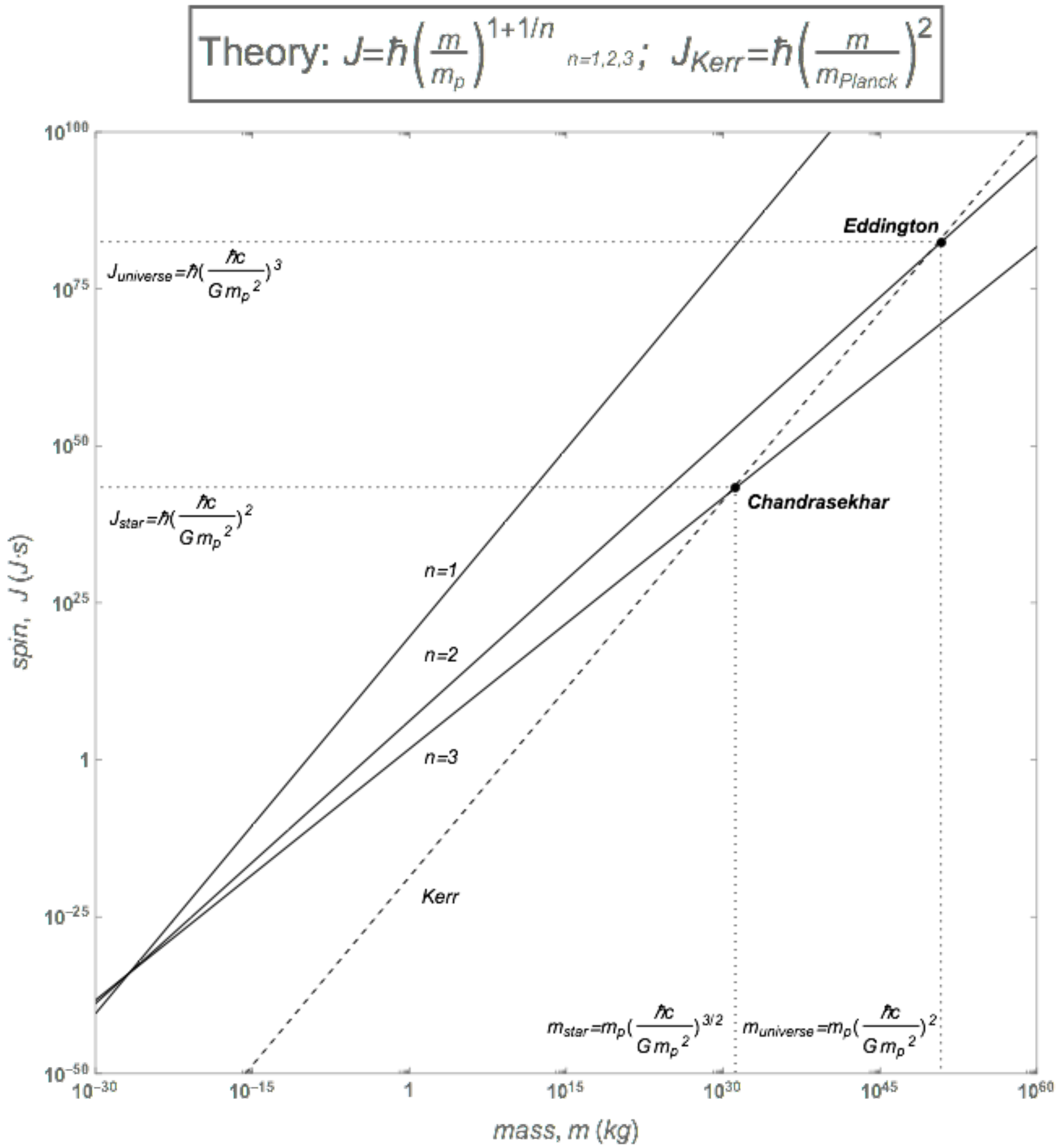


Figure 1. In double logarithmic plot $\log_{10}-\log_{10}$ three Regge spin/mass relations $J^{(1)}(m),\ J^{(2)}(m),\ J^{(3)}(m)$ and Kerr black hole spin/mass are presented. The Kerr $J_{Kerr}(m)$ line is parallel to the Regge trajectory $J^{(1)}(m)$ and intersects $J^{(2)}(m)$ and $J^{(3)}(m)$ in Eddington and Chandrasekhar points with indicated spin and mass coordinates.

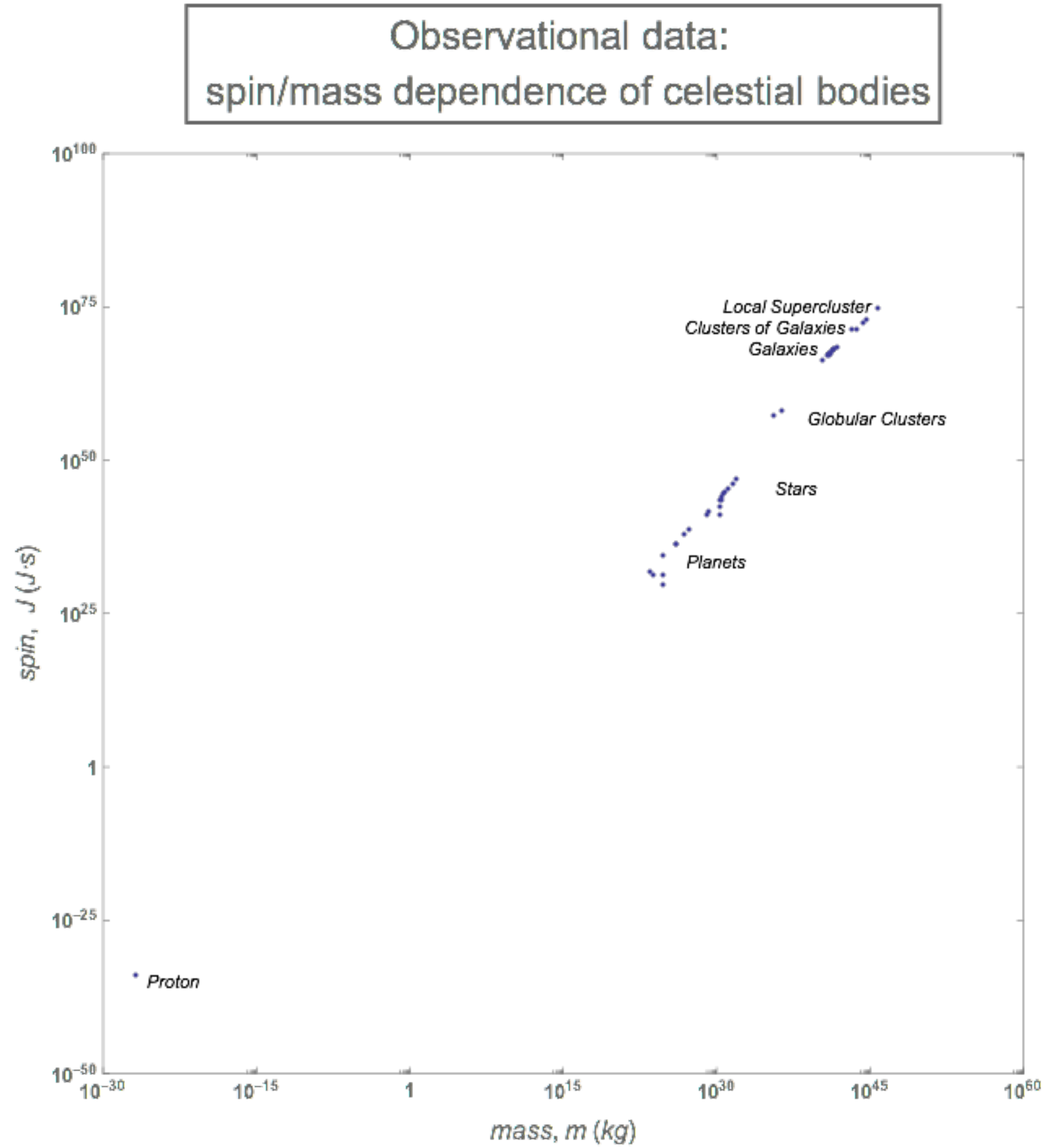


Figure 2. The observational data for planets and stars did not depend on presence or absence of dark matter. For galaxies and clusters of galaxies pre-dark-matter data are used from [11-13]. The *pre-dark matter* data for galaxies are displayed on all our J/m plots. Might there exist a hidden mass in galaxies? Apparently yes, but if this hidden mass participates in gravitational interactions, it could also possess a hidden Regge behaviour.

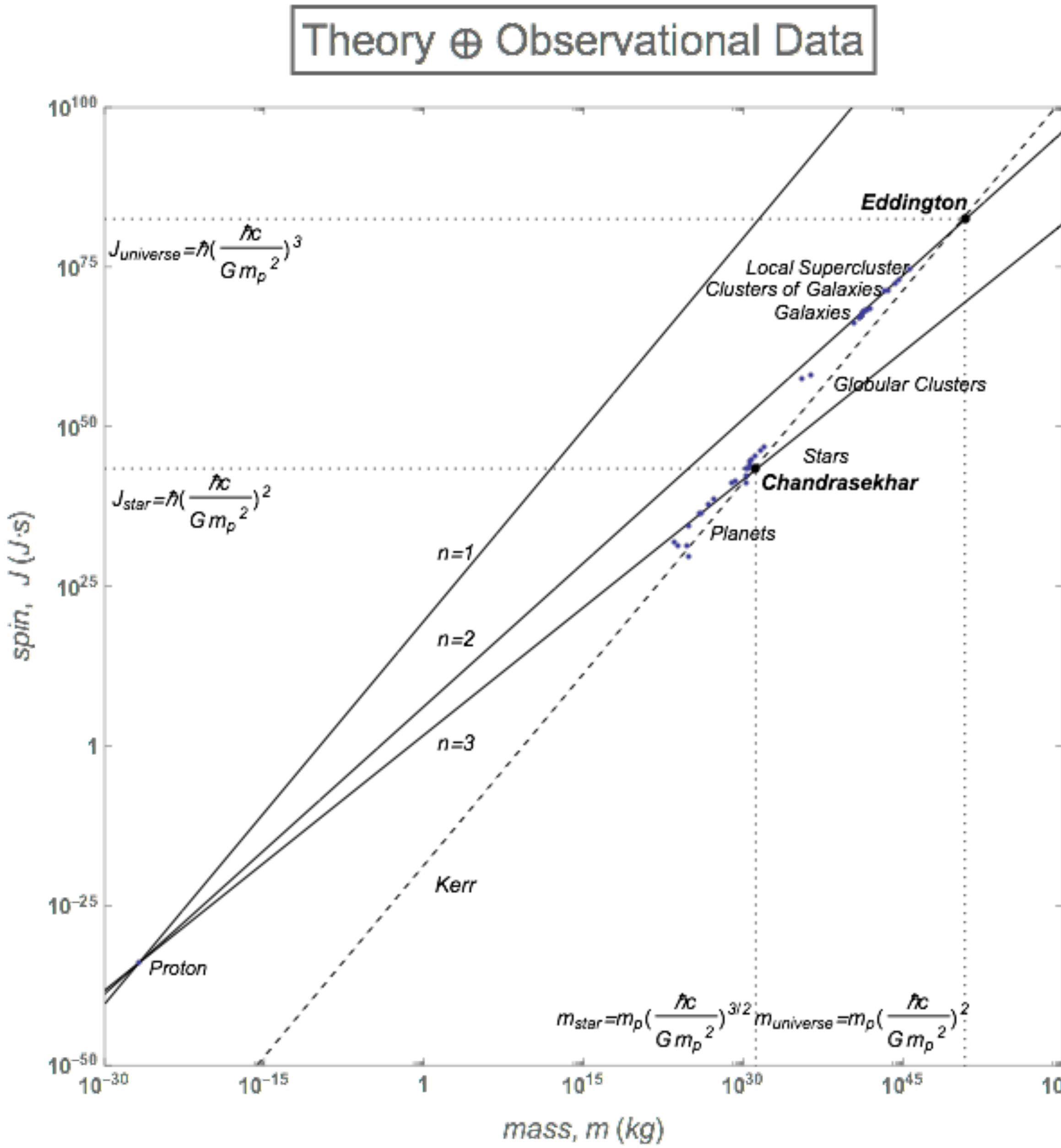


Figure 3. This plot represents the superposition of Figures 1 and 2.

Let us consider some examples of numerical comparison of observational data with the theoretical predictions.

Jupiter. Jupiter is the fastest spinning planet in the Solar System.
It takes 9,925 hours to complete one single rotation around its axis. The mass and spin of Jupiter are well known and are equal:

$$m_{Jupiter} = 1.90 \times 10^{27}\, kg$$
$$J_{Jupiter} = 4.32 \times 10^{38}\, J \cdot s$$

The theoretically calculated spin value neatly coincides with the observed one:

$$J_{Jupiter}^{theor} = \hbar \left(\frac{m_{Jupiter}}{m_p} \right)^{\frac{4}{3}} = 1.25 \times 10^{38}\, J \cdot s$$

<u>*Earth and Earth/Moon system*</u>. The Earth is spinning, turning once on its axis every day, and completing one full turn on its axis during 23.93 hours. The observed mass and spin of the Earth are

$$m_{\oplus} = 5.97 \times 10^{24}\, kg$$
$$J_{\oplus} = 5.91 \times 10^{33}\, J \cdot s$$

The theoretical prediction for Earth's spin

$$J_{\oplus}^{theor} = \hbar \left(\frac{m_{\oplus}}{m_p} \right)^{\frac{4}{3}} = 5.74 \times 10^{34}\, J \cdot s$$

is somewhat larger than the observed value, but closer to the observed total angular momentum of Earth/Moon system

$$J_{\oplus + Moon}^{tot} = 3.47 \times 10^{34}\, J \cdot s$$

Coma Cluster (Abel 1656). Every object in this photo is a galaxy. The Coma Cluster altogether contains 10,000 galaxies.

$$m_{Coma} = 2 \times 10^{14} m_{\odot}$$
$$J_{Coma} = 0.9 \times 10^{73} J \cdot s$$

The theoretical prediction gives close number:

$$J_{Coma}^{theor} = \hbar \left(\frac{m_{Coma}}{m_p} \right)^{\frac{3}{2}} J \cdot s$$

Rotation of the Universe. Recently Michael Longo (U Michigan) analyzing data from Sloan Digital Sky Survey about thousands of spiral galaxies has shown that our Universe has a preferred axis and a net angular momentum. Because of angular momentum conservation this means that the Universe was born spinning. Earlier P. Birch (U Manchester) from the study of position angles and polarization of classical large radio-galaxies demonstrated the existence of a universal vorticity, which means that the Universe is rotating with an angular velocity $\approx 10^{-13} radian / yr$. Using the numerical value of the spin of the *Primeval Hadron,* one can estimate the rotational angular velocity of the Universe. It turns out that $\omega_{Universe} = 10^{-13} radian / yr$, which coincides with Birch's result. The following estimate of the rotational angular velocity of the Universe seems realistic:

$$\omega_{Universe} = 10^{-3 \pm 1} \frac{radian}{age\ of\ Universe}$$

The rotation may play a role of repulsive force mimicking the role of effective $\Lambda - term$ or accelerating dark energy. Hence it can be considered as a substitute of dark energy, as noted by many researchers.

The expression for the angular momentum of the Universe

$$J_{Universe} = \hbar \left(\frac{\hbar c}{G m_p^2} \right)^{\frac{3}{2}}$$

has the interesting consequence

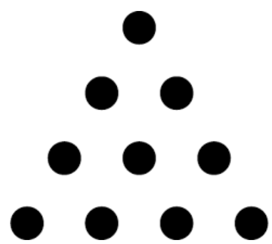
$$\frac{J_{Universe}}{r_{Universe}^3} = \frac{\hbar}{r_p^3} = \sigma$$

which mean that the spin density $\sigma \left([\sigma] = \frac{spin}{volume} \right)$ of the Universe is the same as for the proton. There is the notion that the spin density is the same for all structures, from elementary particles to galaxies and the Universe. The spin density in the ECKS (Einstein-Cartan-Kibble-Siama) theory is related to the torsion Q by $Q = 4\pi G\sigma / c^3$ and, as it is well known, torsion acts

opposite to gravity. Hence it represents a repulsive term and can be considered as a candidate for dark energy.
It is obvious that the exponent in our main formula for spin mass relation $J^{(n)}(m)=m^{1+1/n}\quad n=1,2,3$ exactly coincides with Pythagorean perfect intervals *2* (*octave*), *3/2* (*perfect fifth*), *4/3 (perfect fourth*), indeed

$$1+\frac{1}{n}=\begin{cases}2 & n=1\\ 3/2 & n=2\\ 4/3 & n=3\end{cases}$$

Is this amazing coincidence a manifestation of *Modern Pythagorism*, indoctrinated by Max Planck and recently popularized by Frank Wilczek? Maybe.[3]
Let us point out that *sacred* ratios 2, 3/2, 4/3 are encoded into *Tetractys*' famous Pythagorean symbol



2/1 *Hadrons (octave)*

3/2 *Galaxies (fifth)*

4/3 *Stars (fourth)*

According to Pythagorean tradition the *Tetractys* is a rich transcendent symbol that embraces profoundly deep relationships. By some yet-unknown reason, *Tetractys* encodes diverse physical phenomena. It symbolizes the fundamental numerical ratios that underlies the Universe.

We presented a new, quantum-mechanical model for the origin of the angular momentum of celestial bodies. Unlike the previous classical attempts, our approach gives surprisingly accurate numerical predictions of angular momentum for all spinning astrophysical objects. This occurs for the first time in the history of physics and astronomy. Another outcome from this approach is merely philosophical and it witnesses the unity and simplicity of Nature in micro and macro scales.

ACKNOWLEDGEMENTS
The author wishes to express his gratitude to Victor Matveev and points out special appreciation for light years of invaluable collaboration. The substantial and generous email correspondence with James Bjorken is acknowledged, particularly for his inception of "Bjorken scaling" into physics.

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[3] F. Wilczek, Getting Its From Bits, Nature, v. 397, 303-306, Jan 1999; http://ctp.lns.mit.edu/Wilczek_Nature/Getting%20Its%20from%20Bits.pdf

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APPENDIX

Gmail

rotation

Bjorken, James Daniel Thu, Feb 27, 2014 at 5:22 PM
To: rudolfmm@gmail.com

Dear Rudolf,

I just finished a read of your 1997 paper on rotation. It was total pleasure- - -so clearly written and so interesting. I entered the read with many doubts and questions on my mind. But by the end of the reading most everything had been addressed. A few comments:

At the biggest Big Picture level, your piece is a reminder that rotation is usually an essential complication, not an inessential complication. Since I started learning about gravity and cosmology post-retirement, I have for the most part set rotation aside. (But not completely- - -ten years ago I learned about Kerr black holes and thought quite a bit about them.) You make a very good case that an attitude that dismisses the importance of rotation may be very dangerous.

FRW inflationary cosmology- - -especially its initial condition- - -also sets rotation aside. What is your attitude in this regard? At what point do you join the standard description? (except of course for the rotation of your giant Primeval Hadron?) I have tried to construct my own answer to this question. First of all, dark energy (deSitter space) does not easily talk to rotation. So this suggests that rotation originated post-reheating. The epoch when CP violation entered the scene seems a natural place to introduce it. Then, once in the evolution, the question for me is where in the cosmic mix it is to be located. For me it is most likely to be localized at late times (after matter-radiation equality) in the visible-matter sector, which is the sector which talks to QCD- - -especially to its CP violation. A big problem of course will be to describe in detail the scenario that unfolds after the density contrast becomes of order unity and the cosmic web is created. But things are easier to anticipate in our long-term future. After several efoldings of dark-energy-driven reinflation, there will be islands of matter surrounded by an ocean of dark energy. These islands will have to contain the rotation you describe in Section 6. Trying to visualize the situation may be easier theoretically than dealing with the cosmic-web phenomenology itself.

In the 1990's I was part of a Fermilab experiment. A valued colleague on that effort was Mike Longo (U Michigan). Since his retirement he got interested in looking for the mean helicity of spiral galaxies, and for a while claimed an effect at the 1% level (arXiv 1104.2815). He is a very astute and careful experimentalist. I saw him last fall, and he has moved to other things- - -I got the impression that he of course got criticism and at the end of all that was left with an inconclusive result. But your equation 46 suggests that he got to an intrinsically interesting level and if things could be pushed further out in sensitivity it might be worth another try. Have you looked at this kind of thing? If so, what is your opinion?

Finally, back at the Big Picture level, you remind me that non-rotating Schwarzschild black hole that all theorists (including me) love to death is more extremal than the extremal Kerr black hole. And there is an essential difference in terms of describing the singularity. Andrew Hamilton (U Colorado) is a colleague I have gotten to know in the last few years. He argues forcefully that the presence of an inner horizon makes a huge difference in the description of the insides of a realistic black hole. And I have my own little problem with the Schwarzshild limit, which can be described by the following simple homework problem:

Question: Consider a non-rotating black hole with a mass of order a galactic mass. Drop something in. It falls through the horizon toward the singularity. Assume it is destroyed by tidal forces when those forces become Planckian in scale. This death defines a spacetime event A. Do the same thing a year later; this describes another event B. The interval between A and B is spacelike. What is the distance between A and B? Answer: something like 10^33 cm.

This to me is surprisingly big. But if the geodesic part can somehow be thought of as «helical» the answer might be more intuitive.

Anyway, this reply is getting on the long side. But if nothing else, it is evidence that I really did enjoy your piece.

Regards,

bj