Predictability in Science: Accuracy and Limitations
Pontifical Academy of Sciences, Acta 19, Vatican City 2008
www.pas.va/content/dam/accademia/pdf/acta19/acta19-heller.pdf

PREDICTABILITY, MEASUREMENTS AND COSMIC TIME

MICHAEL HELLER

1. *Introduction*

There exists the common consensus of both physicists and philosophers of science that empirical predictions belong to the core of scientific method. However, the claim that a piano encircles the planet Uranus along an elongated orbit, although empirically – in principle – falsifiable, never would be taken seriously. To define, from the methodological point of view, the nature of scientific predictions and their role in the sciences is not an easy task. For our purposes it is enough to emphasize that any truly scientific prediction (at least as far as physics is concerned) must follow from a theory, expressed in a mathematical form, and must refer to concrete measurement results. For a prediction to be a part of scientific method it is necessary to follow from a scientific theory. Even the most accurate predictions made by an oracle do not count in the sciences. We thus must have a theory *now*, and the prediction *directed to the future*. In this way, the directional flow of time seems to be involved in the very possibility of physics.[1]

And what about retrodictions? If we look at the history of physics, we can easily convince ourselves that retrodictions were as important as predictions. For instance, one of the classical empirical tests for the theory of general relativity, the perihelion motion of Mercury, was very well known to astronomers for a half of century, but this fact did not prevent regarding Einstein's 'prediction' as a major breakthrough in physics. The possibility of reconstructing a state of a physical system in the past (from the present theory) is as important, from the methodological point of

[1] I am treating physics as a typical example of empirical sciences.

view, as predicting it in the future. Only from the psychological point of view we are inclined to attach a greater value to predictions rather than to retrodictions.

It is interesting to notice that these simple remarks on physical methodology lead to a nontrivial cosmological conclusion: *physics, as a science, is possible only in a universe in which exists a local time.* It is clear that it is enough to have a *local* time, i.e., time defined in a neighborhood of the physicists making predictions or retrodictions. In such a time there exist two directions that may be arbitrarily labeled 'the past' and 'the future'. Strictly speaking, the time arrow, pointing to the exactly one direction as to the future, does not seem necessary.[2] Physical time is a time *measured* by a clock. Therefore, physics is possible only in the universe that admits the existence of a clock. One can hardly imagine a clock without it being localized at a certain place. Moreover, to have a *local* time means to have a space-time neighborhood in which a clock is situated. All clocks (and other measuring devices) to be usable by human physicists, must be macroscopic contraptions, or at least must have 'pointers' in the macroscopic world. Thus, the spatio-temporal structures we postulate are macroscopic structures. This, of course, does not exclude the possibility for the physicist to invent and develop theories regarding both the micro-world of atomic and subatomic dimensions, and the world on the cosmic scale, but all these theories have to be tested in our macro-world. As far as the possibility of doing physics is concerned, the macroscopic physics is essential.

The aim of the present paper is to make the above intuitions more precise and to look for some of their philosophical consequences.

2. *Space and Time Measurements*

Among physical measurements especially important are time and space measurements. Although time and space (length) units can be constructed from other physical quantities, space and time are usually regarded as belonging to the most 'primitive' physical magnitudes. According to all theories of macroscopic physics, physical processes unfold on a space-time

[2] This idea was elaborated in my paper: 'The Origins of Time', in: *The Study of Time, IV,* ed. by J.T. Fraser, N, Lawrence and D. Park, Springer, New York, 1981, pp. 90-93, and in the Lecture 4 of the book: *Questions to the Universe. Ten Lectures on the Foundations of Physics and Cosmology,* Pachart, Tuscon, 1986. The present paper is partially based on these works.

arena which, from the mathematical point of view, is a *differential manifold* (or *manifold*, for short). However, the concept of differential manifold as such is too poor a concept to serve as a suitable arena for physics: in the manifold structure there are no conceptual tools that would enable time and space measurements. To acquire such tools, the manifold structure must be enriched by superimposing on it another structure, called *metric structure*. There exist many metric structures and it is up to experiment to decide which of them is correct to model the real world. The present physical paradigm says that it is the *Lorentz* metric structure. More precisely, a four dimensional differential manifold, equipped with the Lorentz metric structure is the mathematical model for physical space-time. Within this model space and time measurements become meaningful operations.

Although in this paper we are essentially interested in the macroscopic space-time model, it is worthwhile to notice that it has been experimentally verified with enormous precision in the realm of microphysics. Predictions of the standard model of elementary particles presupposing this model have been verified with amazing accuracy at length scales of about 10^{-16} cm,[3] and the latest clocks, using a single ion, measure time with the (anticipated) precision of 10^{-18}.[4] We can expect that only below this threshold our manifold model of space-time breaks down. In fact, many works aiming at creating the fundamental theory of physics, predict that something like that should happen.

In the *Introduction* we have said that the minimal cosmological condition for making predictions is the existence of a local time. Geometrically, it is a very tolerant condition. As it is very well known, on every differential manifold a Lorentz metric *locally* always exists. But physics is more than geometry. As we shall see below, the construction of a physical clock requires an interaction of 'nonlocal' regions of space with each other. Moreover, physics regarded as the collection of physical laws, operates not only locally, in our corner of the Universe, but everywhere, i.e., globally. In any case, we should adopt a modern, more universalistic perspective, and at least consider the possibility of the existence of 'other physicists' somewhere in the Cosmos. Since physics can happen on a space-time manifold only if it is a Lorentz manifold, we should look for the necessary and sufficient conditions of the *global* existence of the Lorentz structure.

[3] http://ltp.web.psi.ch

[4] J.C. Bergquist, S.R. Jefferts and D.J. Wineland, 'Time Measurement at the Millenium', *Physics Today,* 54, no 3, 2001, 37.

And in this case the answer is well known. A Lorentz metric *g* exists on a differential manifold *M* if and only if a (smooth) non-vanishing direction field is defined on *M*. In other words, at each point of *M* there should be the possibility to distinguish two directions which we can arbitrarily label 'backward' and 'forward'. If we determine which is 'backward' and which is 'forward' (and if we do that is a smooth way) then the direction field becomes the vector field (to each direction we attach an arrow pointing to the 'forward', say).

The proof of the above theorem is by construction.[5] Let us suppose that on a manifold *M* there exists a Lorentz metric. With the help of this metric, we construct a light cone at every point *p* of *M*. In the interior of every such light cone we choose a vector (which is of course a timelike vector in this metric). This can be done is a smooth way. We thus obtain a smooth vector field on the manifold *M*, and the vector field obviously determines the direction field. And now let us suppose that on *M* there exists a nowhere vanishing direction field. There exists a simple recipe how to construct a Lorentz metric out of this direction field. This can be done in such a way that the direction field becomes timelike in this metric.

This result is striking. It can be interpreted by saying that a differential manifold is a suitable arena for physics if and only if every observer in it can distinguish two time directions. Perhaps the term 'observer' is here an exaggeration. Such an observer need not to have a Ph.D. in physics; it is enough for him/her to be equipped with a suitable feeling of time. In this sense, time is a precondition for physics.

This result should be understood correctly. The above theorem asserts that on a manifold *M* there exists a Lorentz metric globally if and only if on *M* there exists a nowhere vanishing direction field that can be interpreted as a sort of time feeling by every observer. The Lorentz metric exists globally, but it is enough for the required time to be local. Clocks carried by all local observers need not be synchronized. The only requirement is that two time directions change smoothly from one observer to another (so as the non-vanishing direction field be smooth). Of course, we can postulate the existence of a global time on the manifold *M*, but this requires an additional condition imposed on it. Surprisingly enough, this condition is also related to the possibility of doing physics on *M*.

[5] For details see: R. Geroch and G.T. Horowitz, 'Global Structure of Spacetimes', in: *General Relativity. An Einstein Centenary Survey,* ed. by S.W. Hawking, W. Israel, Cambridge University Press, Cambridge, 1979, pp. 212-293, especially pp. 218-220.

3. *Global Time and Stable Measurements*

Everyone who ever had to do something with performing physical measurements knows that they give results only within certain limits of accuracy (even without taking into account quantum indeterminacies). And such measurements give us valuable information about the world on which all of our science is based. This very fact also contains an information about the structure of the Universe. Imagine a 'malicious universe' in which even the slightest change in the measurement result leads to drastically different physical theories. In such a universe no theory could ever be empirically verified or falsified. Small errors inherent in every measurement would spoil the effectiveness of the empirical method. Since this is obviously not the case in our Universe, we must acknowledge that it possesses a certain stability property: small changes in the measurement results require only small adjustments in a theory that predicts these results.

In particular, the same is true as far as space and time measurements are concerned. But it is Lorentz metric that is responsible for them. Consequently, we must ascribe to it the corresponding stability property. We should postulate that small changes in a given Lorentz metric should not produce drastic changes in the (global) space-time structure. In particular, we should postulate that small changes in a given Lorentz metric should not produce closed timelike curves (provided that such curves were absent in the original space-time). The absence of closed timelike curves is evidently related to causality: following such a curve an observer could kill his father before his birth. Therefore, space-times containing no closed timelike curves are justly called *causal* space-times, and any such space-time in which a small perturbation of its Lorentz metric does not produce closed timelike curves is called *stably causal*. If this condition is satisfied, the space-time is not only causal, but also causal with a certain margin of safety, it is not on the verge of violating causality.

And now the surprise. There is a theorem due to Hawking which asserts that space-time is stably causal if and only if it admits a global time.[6] Therefore, if the temporal properties of the world are improved, so as the clocks of local observers indicate the same time flow, then the stability of space and time measurements is automatically guaranteed. To give Hawking's theorem its precise meaning we must determine the meaning of the 'global time' in this context.

[6] S.W. Hawking, 'The Existence of Cosmic Time Functions', *Proc. Roy. Soc.* London A 308, 1968, 433-435.

To extract the geometric meaning from the statement that an observer carries a clock is equivalent to saying that there exists a smooth monotonically increasing function defined along the timelike curve that is the history of this observer in space-time. The real values of this function are interpreted to be indications of the clock carried by the observer. Or, in other words, if an observer carries a clock, its indications associate a real number with each point along observer's history. This defines a real, monotonically increasing function along this history. The global time means that the same function is defined along every timelike curve in space-time. And the Hawking theorem asserts that this is equivalent to the stable causality of space-time.

One more caveat. The existence of global time in the above sense does not presuppose the existence of the universal 'surface of simultaneity': the clocks of all observers indicate the same time flow, but they need not be synchronized. To guarantee such a possibility would require further strengthening causal properties of space-time.[7] But this is another story.

4. *Clocks in the Universe*

So far all our arguments were purely geometric. For instance, we have identified a clock carried by a local observer with a smooth monotonically increasing function along a suitable timelike curve. But any real clock is a physical contraption requiring certain conditions for its construction and functioning. A physical clock is a subsystem of the world, the changes of which could be used to compare them with changes of the environment and to 'monitor' them. If the world were too simple to admit such a subsystem, or too chaotic to allow for its predictable behavior, no clock could be constructed in it. The world in the state of thermodynamic equilibrium, in which only random motions of atoms are possible, is an example of such a 'clockless' system. As noticed by Lee Smolin: 'A world with a clock is then one that is organized to some extent; it is a world somewhere on the boundary between chaos and stasis. The world must be sufficiently dynamical that there is no danger of reaching equilibrium, after which it is chaotic at the microscopic level and static on all larger scales. But it must be organ-

[7] The corresponding space-time should be glabally hyperbolic; see: S.W. Hawking and G.R.S. Ellis, *The Large Scale Structure of Space-Time,* Cambridge University Press, Cambridge, 1973, pp. 206-212.

ized enough that distinct subsystems may be identified that preserve enough order to evolve predictably and simply'.[8]

Every clock, by its very nature, is a cosmological device. 'The point is – writes David Park – that a simple kitchen clock, just as much as the great cosmological models of antiquity, registers the pulse of the universe and keeps time with it, for the same physical laws govern both of them'.[9]

5. *Why Everything Does not Happen at Once*

It was Whitrow who wrote: 'any theory which endeavours to account for time completely ought to explain why it is that everything does not happen at once'.[10] This statement encapsulates intuitions underlying all geometric theorems and their interpretations presented in this paper. In a universe, in which everything happened at once, physics would be trivial as an empty set, and predictions would be impossible by definition. Doing physics presupposes a temporal extension of the world for a dynamics to develop and predictions to be made and verified.

The Universe is a structure which is accessible to us through its various aspects. These aspects, however, are not independent of each other, and some of them are more fundamental than the others. As we have seen, there are strong reasons to think that temporal aspects of the world belong to the most fundamental ones. Some degree of temporality is necessary for the world to be a physical world.

[8] L. Smolin, *The Life of the Cosmos,* Oxford University Press, New York – Oxford, 1997, pp. 287-288.

[9] D. Park, *The Image of Eternity. Roots of Time in the Physical World,* The University of Massachusetts Press, Amherst, 1980, p. 39.

[10] G.J. Whitrow, *The Nature of Time*, Penguin Books, 1975, p. 132.